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*Research Article*

# Classification of All Single Traveling Wave Solutions of (3 + 1)-Dimensional Jimbo-Miwa Equation with Space-Time Fractional Derivative

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In this paper, the complete discrimination system method is used to construct the single traveling wave solutions for the $(3+1)$-dimensional Jimbo-Miwa equations with space-time fractional derivative. As a result, we get the exact traveling wave solutions of the $(3+1)$-dimensional Jimbo-Miwa equation with space-time fractional derivative, which include rational function solutions, Jacobian elliptic function solutions, hyperbolic function solutions, and trigonometric function solutions. Some graphical representations of the solutions are also provided. Finally, the obtained solution is compared with the existing literature.

## 1. Introduction

In recent decades, a large number of nonlinear partial differential equation (NLPDE) have been proposed and studied in order to describe nonlinear wave phenomena in the fields of hydrodynamics, plasma physics, solid physics, and condensed matter physics. Moreover, in order to make these NLPDE better fit the actual situation, many researchers also simulate complex factors by using mathematical tools, such as stochastic differential and fractional differential [1–5]. In the research methods, traveling wave solutions, as a special kind of analytical solutions of NLPDE, plays an important role in understanding nonlinear wave phenomena [6–12]. Therefore, the exact traveling wave solution is an attractive work in the study of theory and practice.

In 1983, Jimbo and Miwa [13] firstly proposed the following nonlinear partial differential equation in the study of Lie algebra:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, p, q, r, s \in R, \quad (1)$$

which is called classical $(3+1)$-dimensional Jimbo-Miwa (JM) equation. JM equation is the second equation of the famous Kadomtsev–Petviashvili (KP) hierarchy of integrated systems. It plays an important role in the study of three-dimensional waves in plasma and optics.

Although JM equation is nonintegrable, it has solitary wave solutions, and the behavior of these solitary waves is different from that of solitons in multiple collisions, which has attracted much attention to its traveling wave solutions. In 2000, the generalized tanh method was used [14] to obtain some traveling wave solitary wave solutions. In 2001, some hyperbolic function solutions and trigonometric function solutions of JM equation were obtained by the homogeneous balance method [15]. In 2007, through the double soliton method and bilinear method, Dai et al. [16] obtained some special traveling wave solutions, including smooth cross kink wave solution, singular periodic kink wave solution, singular periodic soliton wave solution, and singular double periodic wave solution. In 2017, Wazwaz obtained multiple-soliton solutions using the tanh-coth method [17]. In 2010, Song and Ge use G′/G expansion method; three new traveling wave solutions were expressed as hyperbolic function, trigonometric function, and rational function in [18]. In 2011, Li et al. [19] applied the generalized three wave method to derive accurate three wave solutions, including periodic cross kink wave solutions, double periodic solitary wave solutions, and solitary wave solutions. In 2016, Ma gets four kinds of block solutions for Hirota bilinear form which are given in [20]. In 2017, Su and Dai [21] gave its single-periodic wave solution and double-periodic wave solution by using the multidimensional elliptic function. It is worth mentioning that with the development of traveling wave solution theory, more and more effective methods are applied to find traveling wave solutions. Very recently, $N$-soliton solutions to integrable equations have systematically been studied by the Hirota direct method, see [22–25].

Due to various real-world applications of fractional differential equations [26–30], we consider the $(3+1)$-dimensional JM equation with fractional order space-time derivatives in the following form [31]:

$$\begin{aligned} &2D_y^{\beta}D_t^{\alpha}u + 3D_y^{\beta}uD_x^{2\eta}u + 3D_x^{\eta}uD_x^{\eta}D_y^{\beta}u \\ &\quad - 3D_x^{\eta}D_z^{\gamma}u + D_x^{3\eta}D_y^{\beta}u \\ &= 0, 0 < \alpha, \beta, \gamma, \eta < 1, \end{aligned} \tag{2}$$

where $\alpha, \beta, \gamma$, and $\eta$ are denoted the order of fractional derivative.

The fractional Jimbo-Miwa (FJM) equation contains the same property of fractional KP equation and fractional KdV equation. Also, it practically describes the $(3+1)$-dimensional travelling wave nature [32]. The exact analytical solutions of $(3+1)$ space-time FPDEs are very difficult to handle, due to the presence of very complicated nonlinear terms. Because of that, numerous numerical and analytical methods have been suggested for getting solutions to those types of equations. The analytical solutions of conformable time-fractional space-time fractional JM equation have been presented by Korkmaz [32]. In 2017, the exp(-(phi)) method is used to construct the exact solutions of nonlinear space-time fractional $(3+1)$-dimensional Jimbo-Miwa equation [33]. In 2019, Zhou et al. [34] applied the bifurcation method of dynamical system to investigate the phase space geometry of $(3+1)$-dimensional JM equation. In 2020, Sahoo and Ray [35] applied extended G′/G-expansion method to space-time fractional $(3+1)$-dimensional JM equation and got antikink wave solutions.

The paper is constituted in six sections as described as the following: the introduction of local fractional calculus and algorithm of the complete discrimination system method have been described in Section 2. We simplify Equation (2) to the nonlinear ordinary differential equation by fractional traveling wave transformation in Section 3. The classification of all single traveling wave solutions has been presented in Section 4. The numerical simulation of results have been presented in Section 5. A precise conclusion of the presented work has been presented in Section 6.

## 2. Introductions of Local Fractional Calculus and Algorithm of the Proposed Method

### 2.1. Definition of Local Fractional Derivative

*Definition 1.* Let $h(x) \in C_{\alpha}(m, n)$. Then, the derivative with local fractional-order $\alpha$ at $x = x_0$ for $h(x)$ is presented as [36]:

$$h^{(\alpha)}(x_0) = \left.\frac{d^{\alpha}h(x)}{dx^{\alpha}}\right|_{x=x_0} = \lim_{x \longrightarrow x_0} \frac{\Delta^{\alpha}(h(x) - h(x_0))}{(x - x_0)^{\alpha}}, \tag{3}$$

where $\Delta^{\alpha}(h(x) - h(x_0)) \cong \Gamma(1+\alpha)(h(x) - h(x_0))$ and $0 < \alpha \le 1$.

*Remark 2.* The following rule holds:

$$\frac{d^{\eta}x^{k\eta}}{dx^{\eta}} = \frac{\Gamma(1+k\eta)}{\Gamma(1+(k-1)\eta)}x^{(k-1)\eta}. \tag{4}$$

*Remark 3.* If $\omega(x) = (f \circ g)(x)$, where $g(x) = u(x)$, then we get

$$\frac{d^{\eta}\omega(x)}{dx^{\eta}} = f^{(\eta)}(u(x))\left(u^{(1)}(x)\right)^{\eta}, \tag{5}$$

when $f^{(\eta)}(u(x))$ and $u^{(1)}(x)$ exist.

If $\omega(x) = (f \circ g)(x)$, where $g(x) = u(x)$, then we get

$$\frac{d^{\eta}\omega(x)}{dx^{\eta}} = f^{(1)}(u(x))u^{(\eta)}(x), \tag{6}$$

when $f^{(1)}(u(x))$ and $u^{(\eta)}(x)$ exist.

*2.2. The Algorithm of the Complete Discriminant System Method.* The complete discriminant system method was first introduced by Yang and his team members in 1961. The higher-order polynomial discriminant system established by this method can be used to find the traveling wave solutions of fractional nonlinear fractional partial differential equations. The primary steps are given as follows.

*Step 1.* Here, we have considered a nonlinear fractional differential equation as the following form

$$\begin{aligned} &P\left(u, D_t^{\alpha}u, D_x^{\eta}u, D_y^{\beta}u, D_z^{\gamma}u, D_t^{\alpha}D_t^{\alpha}u, D_t^{\alpha}D_x^{\eta}u, D_x^{\eta}D_y^{\beta}u, \cdots\right) \\ &\quad = 0, 0 < \alpha, \beta, \gamma, \eta < 1, \end{aligned} \tag{7}$$

where $D_x^{\eta}u, D_y^{\beta}u, D_z^{\gamma}u$, and $D_t^{\alpha}u$ are the local fractional derivatives of $u$ with respect to $x, y, z$, and $t$. $P$ is a polynomial of $u = u(x, y, z, t)$ and its various partial derivatives, in which the highest-order derivatives and nonlinear terms are involved.

*Step 2.* The fractional complex transform is presented as (see [10, 11])

$$u(x,y,z,t)=U(\xi),\xi=\frac{k_1x^{\eta}}{\Gamma(1+\eta)}+\frac{k_2y^{\beta}}{\Gamma(1+\beta)}+\frac{k_3z^{\gamma}}{\Gamma(1+\gamma)}-\frac{\nu t^{\alpha}}{\Gamma(1+\alpha)}, \tag{8}$$

where $k_1, k_2, k_3$, and $\nu\neq 0$ are arbitrary constants.

By using the chain rule (see Remark 3), we have:

$$D_t^{\alpha}u=-\nu\sigma_t U_{\xi'}, D_x^{\eta}u=k_1\sigma_x U_{\xi'},\cdots. \tag{9}$$

Here, $\sigma_t, \sigma_x$ are the fractal indexes, without loss of generality, $\sigma_t=\sigma_x=\theta$, $\theta$ is a constant.

Equation (7) is reduced to the following nonlinear ordinary differential equation (ODE) by using Equation (8):

$$P\left(U,-\nu\theta U', k_1\theta U', k_2\theta U', k_3\theta U', \nu^2\theta^2U'',-k_1\nu\theta^2U'',\cdots\right)=0. \tag{10}$$

Without losing generality, Equation (8) can also be written in the following form:

$$Q\left(U,kU',k^2U'',k^3U''',\cdots,\nu U',\cdots\right)=0, \tag{11}$$

where $Q$ is a polynomial in $u$ and its derivatives and notation $(')$ are the derivatives with respect to $\xi$.

*Step 3.* Rewrite Equation (11) into the following form:

$$\left[U'(\xi)\right]^2=G(U,\theta_1,\theta_2,\cdots,\theta_m), \tag{12}$$

where $\theta_1,\theta_2,\cdots,\theta_m$ are parameters.

*Step 4.* Integrate both sides of Equation (12) once, we have

$$\pm(\xi-\xi_0)=\int\frac{1}{\sqrt{G(U,l_1,l_2,\cdots,l_m)}}dU, \tag{13}$$

where $G(U)$ is a polynomial function. In this paper, $G(U)$ is a third-degree polynomial in the form

$$G(U)=U^3+l_2U^2+l_1U+l_0, \tag{14}$$

where $l_0, l_1, l_2$ are constants with respect to the parameters $\theta_1,\theta_2,\cdots,\theta_m$.

According to the complete discrimination system (23) for the third-degree polynomial, the roots of $G(U)$ can be classified, and then, the solution of Equation (13) can be obtained. The detailed classification will be given in Section 3.

## 3. The Fractional Traveling Wave Transformation for Space-Time FJM Equation

This section contains the solution of Equation (2) by using the complete discriminant system method.

By the help of Equation (8), Equation (2) can be reduced to the following third-order nonlinear ODE:

$$(2\nu k_2+3k_1k_3)U'-3k_1^2k_2\left(U'\right)^2-k_1^3k_2U'''=C_1. \tag{15}$$

Multiply both sides of (15) by U′′ and integrate once, we get

$$\frac{1}{2}(2\nu k_2+3k_1k_3)\left(U'\right)^2-k_1^2k_2\left(U'\right)^3-\frac{1}{2}k_1^3k_2\left(U''\right)^2=C_1U'+C_0, \tag{16}$$

where $C_0$ and $C_1$ are integral constants.

Let $U'(\xi)=V(\xi)$. Then, Equation (16) can be written as

$$\left(V'\right)^2=-\frac{2}{k_1}V^3+\frac{2\nu k_2+3k_1k_3}{k_1^3k_2}V^2-\frac{2C_1}{k_1^3k_2}V-\frac{2C_0}{k_1^3k_2}. \tag{17}$$

Take a suitable transform in the following form:

$$\begin{cases} V(\xi)=\left(-\frac{2}{k_1}\right)^{-1/3}W(\xi_1), & \xi_1=\left(-\frac{2}{k_1}\right)^{1/3}\xi, \\ d_2=\left(-\frac{2}{k_1}\right)^{-2/3}\frac{2\nu k_2+3k_1k_3}{k_1^3k_2}, & d_1=\left(-\frac{2}{k_1}\right)^{-1/3}\frac{2C_1}{k_1^3k_2}, d_0=-\frac{2C_0}{k_1^3k_2}. \end{cases} \tag{18}$$

Substituting (18) into Equation (17), we get a nonlinear ODE:

$$\left(W_{\xi_1}{}'\right)^2=W^3+d_2W^2+d_1W+d_0. \tag{19}$$

Assume that

$$f(W)=W^3+d_2W^2+d_1W+d_0. \tag{20}$$

Then, we write Equation (19) to the form:

$$\frac{\mathrm{dW}}{\sqrt{\mathrm{f(W)}}}=\pm\mathrm{d}\xi_1=\pm\left(-\frac{2}{\mathrm{k}_1}\right)^{1/3}\mathrm{d}\xi. \tag{21}$$

Equation (19) can be changed to the following integral form by using Equation (21):

$$\pm\left(-\frac{2}{k_1}\right)^{1/3}(\xi-\xi_0)=\int\frac{dW}{\sqrt{f(W)}}, \tag{22}$$

where $\xi_0$ is an integral constant.

The corresponding complete discrimination system of (19) is

$$\begin{cases} \Delta & = -27\left(\dfrac{2d_2^3}{27} + d_0 - \dfrac{d_1 d_2}{3}\right)^2 - 4\left(d_1 - \dfrac{d_2^3}{3}\right)^3, \\ D_1 & = d_1 - \dfrac{d_2^2}{3}. \end{cases} \quad (23)$$

However, in this study, we aim to establish new exact solitary wave solutions to the space-time fractional (3 + 1 )-dimensional JM Equation (2) by complete discriminant system method with the aid of symbolic computation software Maple. A kind of comparison analysis will be provided alongside the results and discussion of the considered problems. Some graphical representations of the problems and that of comparison will be provided at the end.

## 4. The Classification of All Single Traveling Wave Solutions

*Case 1.* If $\Delta = 0$, and $D_1 < 0$, then $f(W) = 0$ has a double real root and a single real root. Denote $f(W) = (W - r_1)^2(W - r_2)$, where $r_1 \neq r_2$.

When $W > r_2$, we have

$$\begin{aligned} &\pm\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0) \\ &= \int \frac{1}{\sqrt{(W - r_1)^2(W - r_2)}} dW \\ &= \begin{cases} \dfrac{1}{\sqrt{r_1 - r_2}} ln\left|\dfrac{\sqrt{W - r_2} - \sqrt{r_1 - r_2}}{\sqrt{W - r_2} + \sqrt{r_1 - r_2}}\right|, & r_1 > r_2, \\ \dfrac{2}{\sqrt{r_2 - r_1}} \arctan\sqrt{\dfrac{W - r_2}{r_2 - r_1}}, & r_1 < r_2. \end{cases} \end{aligned} \quad (24)$$

Then, by $V(\xi) = (-(p+q)/3)^{-1/3} W((-(p+q)/3)^{1/3}\xi)$ and (24), the solution of Equation (17) is

$$\begin{aligned} V_1(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} &\left[r_2 + (r_1 - r_2) \tan \text{h}^2 \frac{1}{2}\right. \\ &\left.\cdot \left(\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0)\right)\right], r_1 > r_2, \end{aligned} \quad (25)$$

$$\begin{aligned} V_2(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} &\left[r_2 + (r_1 - r_2) \cot \text{h}^2 \frac{1}{2}\right. \\ &\left.\cdot \left(\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0)\right)\right], r_1 > r_2, \end{aligned} \quad (26)$$

$$\begin{aligned} V_3(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} &\left[r_2 + (r_2 - r_1)\tan^2 \frac{\sqrt{r_2 - r_1}}{2}\right. \\ &\left.\cdot \left(\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0)\right)\right], r_1 < r_2. \end{aligned} \quad (27)$$

We can see that when $\Delta = 0$ and $D_1 < 0$, Equation (17) has solitary wave solutions (25) and (26) and has trigonometric function periodic solutions (27).

*Case 2.* If $\Delta = 0$, and $D_1 = 0$, then $f(W) = 0$ has a triple real root. Denote $f(W) = (W - r)^3$, we have

$$\pm\left(\frac{p+q}{3}\right)^{1/3}(\xi - \xi_0) = \int \frac{1}{\sqrt{(W - r)^3}} dW. \quad (28)$$

Then, the solution of Equation (17) is

$$V_4(\xi) = 4\left(-\frac{2}{k_1}\right)^{-2/3}(\xi - \xi_0)^{-2} + \left(-\frac{2}{k_1}\right)^{-1/3} r. \quad (29)$$

Equation (29) shows that the factional JME (17) has rational function solution.

*Case 3.* If $\Delta > 0$, and $D_1 < 0$, then $f(W) = 0$ has three different real roots, $r_1, r_2, r_3$, and $r_1 < r_2 < r_3$. If $r_1 < W < r_3$, taking the transformation $W = r_1 + (r_2 - r_1) \sin^2\zeta$, then we obtain

$$\pm\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0) = \frac{2}{\sqrt{r_3 - r_1}} \int \frac{1}{\sqrt{1 - m_1^2 sin^2\zeta}} d\zeta, \quad (30)$$

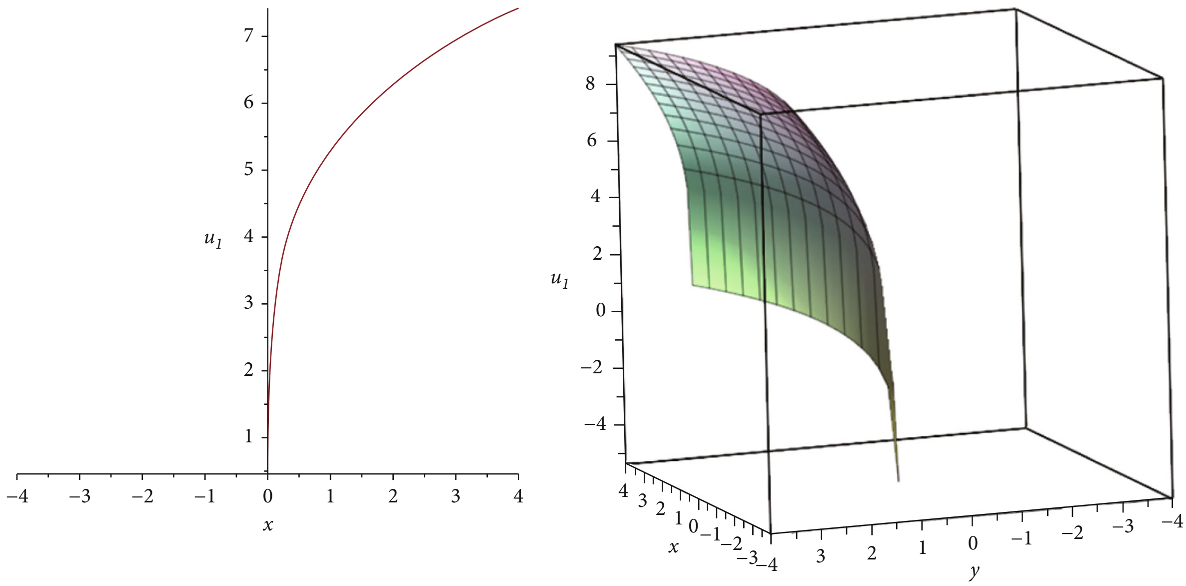
where $m_1^2 = (r_2 - r_1)/(r_3 - r_1)$.

By the definition of Jacobi function and (30), we have

$$W = r_1 + (r_2 - r_1) sn^2\left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0), m_1\right). \quad (31)$$

Thus, the solution of Equation (17) is

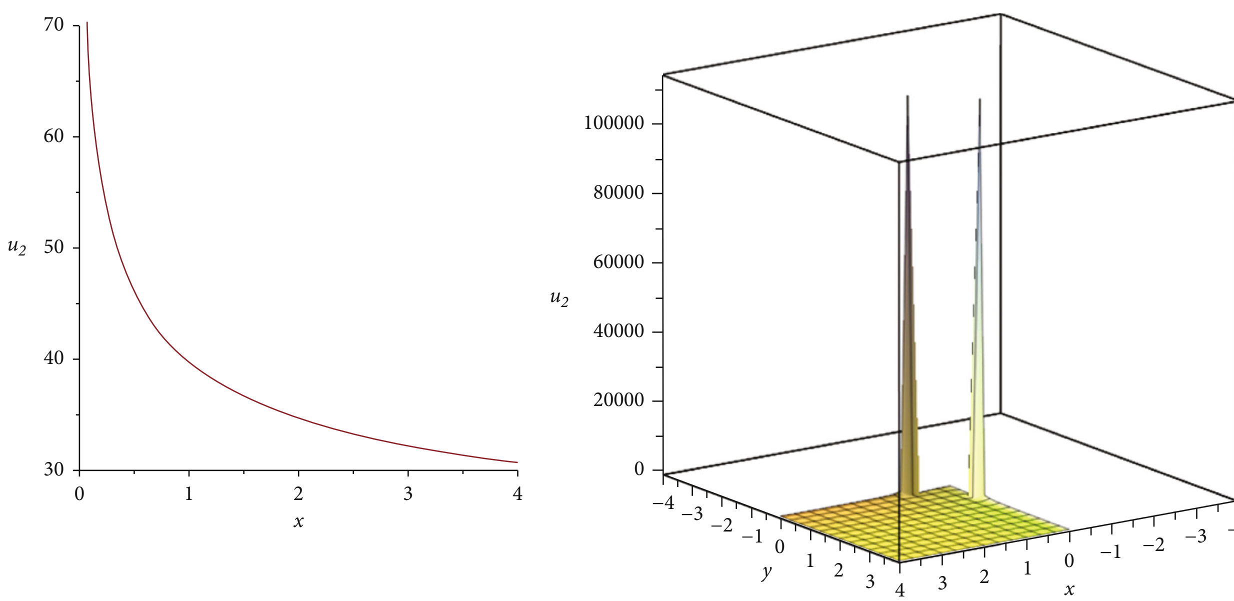
$$\begin{aligned} V_5(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} &\left[r_1 + (r_2 - r_1) sn^2\right. \\ &\left.\cdot \left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0), m_1\right)\right]. \end{aligned} \quad (32)$$




(a) $u_1(x, y, z, t)$ when $y = 1, z = 0, t = 1$

(b) $u_1(x, y, z, t)$ when $z = 0, t = 1$

FIGURE 1: $u_1(x, y, z, t)$ when $k_1 = k_2 = k_3 = \nu = \alpha = \beta = \gamma = \eta = 1/4$, $c_0 = c_1 = 0$, $\xi_0 = 0$.




(a) $u_2(x, y, z, t)$ when $y = 1, z = 0, t = 1$

(b) $u_2(x, y, z, t)$ when $z = 0, t = 1$

FIGURE 2: $u_2(x, y, z, t)$ when $k_1 = k_2 = k_3 = \nu = \alpha = \beta = \gamma = \eta = 1/4$, $c_0 = c_1 = 0$, $\xi_0 = 0$.

If $W > r_3$, take the transformation $W = (-r_2 \sin^2\zeta + r)/(\cos^2\zeta)(/)$; the solution of Equation (17) is

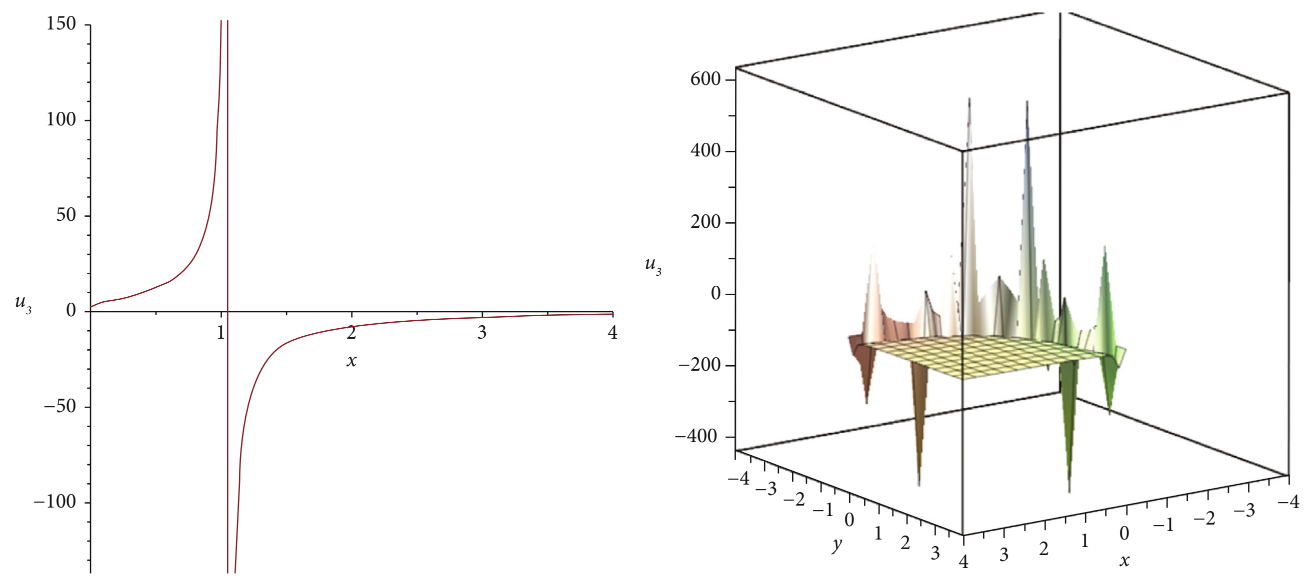
$$V_6(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} \left[\frac{r_3 - r_2 sn^2\left(\sqrt{r_3 - r_1}/2(-2/k_1)^{1/3}(\xi - \xi_0), m_2\right)}{cn^2\left(\sqrt{r_3 - r_1}/2(-2/k_1)^{1/3}(\xi - \xi_0), m_2\right)}\right], \tag{33}$$

where $m_2^2 = (r_2 - r_1)/(r_3 - r_1)$.

So, we get two periodic solutions, because sn and cn are biperiodic functions. Note that

$$\lim_{k \longrightarrow 1} \mathrm{sn}(x, k) = \tanh(x), \lim_{k \longrightarrow 1} \mathrm{cn}(x, k) = \mathrm{sech}(x). \tag{34}$$

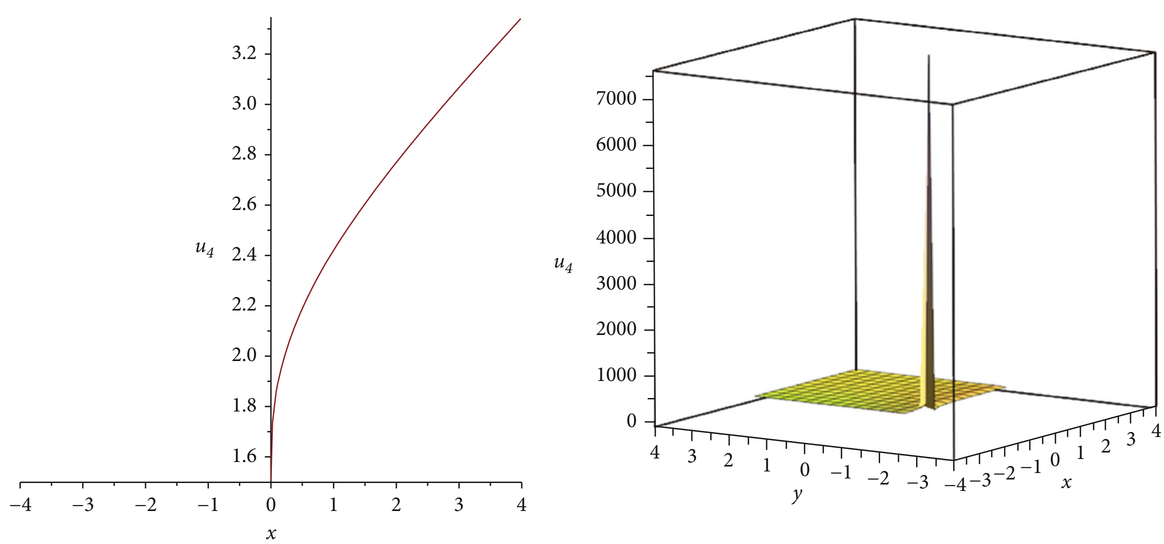
If $r_2 \longrightarrow r_3$, $m \longrightarrow 1$, then Equation (17) has two solitary wave solutions




(a) $u_3(x, y, z, t)$ when $y = 1, z = 0, t = 1$

(b) $u_3(x, y, z, t)$ when $z = 0, t = 1$

FIGURE 3: $u_3(x, y, z, t)$ when $k_1 = k_2 = k_3 = \nu = \alpha = \beta = \gamma = \eta = 1/4$, $c_0 = c_1 = 0$, $\xi_0 = 0$.




(a) $u_4(x, y, z, t)$ when $y = 1, z = 0, t = 1$

(b) $u_4(x, y, z, t)$ when $z = 0, t = 1$

FIGURE 4: $u_4(x, y, z, t)$ when $k_1 = k_3 = \nu = \alpha = \beta = \gamma = \eta = 1/4$, $k_2 = -3/8$, $c_0 = c_1 = 0$, $\xi_0 = 0$.

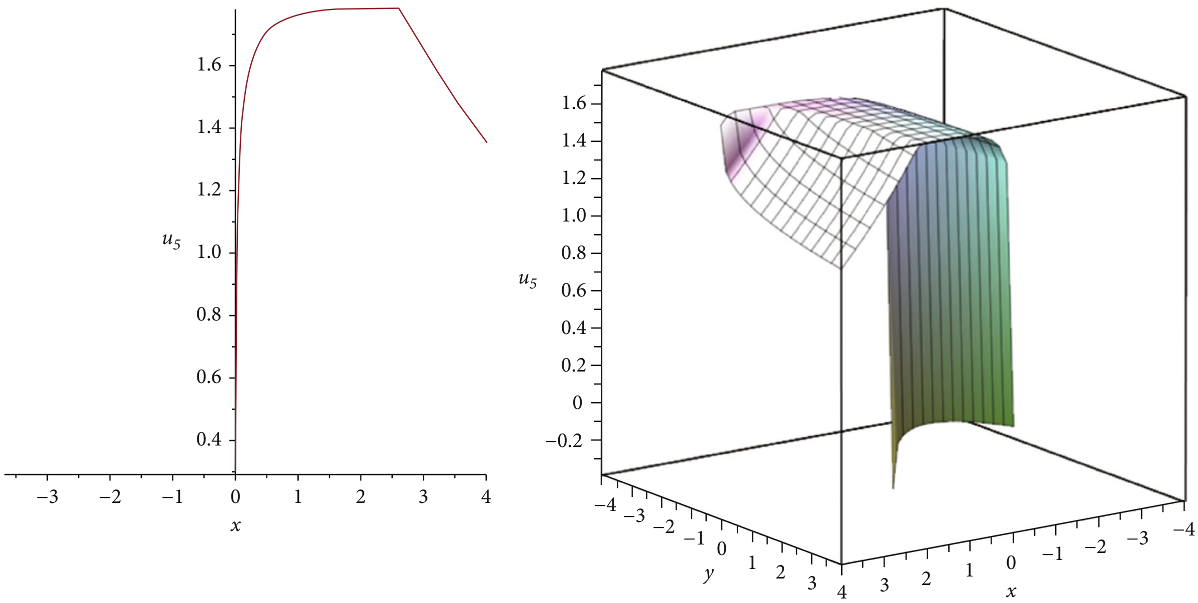
$$v_5(\xi_1) = \alpha_1 + (\alpha_2 - \alpha_1)\tanh^2\left(\frac{1}{2}\sqrt{\alpha_3 - \alpha_1}(\xi_1 - \xi_0)\right),$$
$$v_6(\xi_1) = \frac{-\alpha_2 \tanh\left(1/2\sqrt{\alpha_3 - \alpha_1}(\xi_1 - \xi_0)\right) + \alpha_3}{\operatorname{sech}\left(1/2\sqrt{\alpha_3 - \alpha_1}(\xi_1 - \xi_0)\right)}. \tag{35}$$

*Case 4.* If $\Delta < 0$, then $f(W) = 0$ has only one real roots. Denote $f(W) = (W - r)(W^2 + pW + q)$, where $r^2 - 4q < 0$.

If $W > r$, taking the transformation $W = r + \sqrt{r^2 + pr + q}\tan^2\zeta/2$, then we obtain

$$\begin{aligned}
&\pm\left(-\frac{2}{k_1}\right)^{1/3}(\xi - \xi_0) \\
&= \int \frac{1}{\sqrt{(W - r)(W^2 + pW + q)}}\,dW \\
&= \int \frac{\sqrt{r^2 + pr + q}\tan\zeta/2\cos^2\zeta/2}{(r^2 + pr + q)^{3/4}\tan\zeta/2\cos^2\zeta/2\sqrt{1 - m_3^2\sin^2\zeta}}\,d\zeta \\
&= \frac{2}{(r^2 + pr + q)^{1/4}}\int \frac{1}{\sqrt{1 - m_3^2\sin^2\zeta}}\,d\zeta,
\end{aligned} \tag{36}$$

where $m_3^2 = 1/2(1 - ((r + r/2)/\sqrt{r^2 + pr + q}))$.




(a) $u_5(x, y, z, t)$ when $y = 1, z = 0, t = 1$

(b) $u_5(x, y, z, t)$ when $z = 0, t = 1$

FIGURE 5: $u_5(x, y, z, t)$ when $k_1 = 2, k_2 = k_3 = \nu = \alpha = \beta = \gamma = \eta = 1/4, c_0 = 0, c_1 = 1, \xi_0 = 0$.

From the definition of Jacobi function and (36), we have

$$\cos \zeta = \frac{2\sqrt{r^2 + pr + q}}{W - r + \sqrt{r^2 + pr + q}} - 1. \tag{37}$$

Then, if $W > r$, by Equation (37), the solution of Equation (17) is

$$V_7(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} \left[\frac{2\sqrt{r^2 + pr + q}}{1 + cn\left(\left(r^2 + pr + q\right)^{1/4} \left(-2/k_1\right)^{-1/3} (\xi - \xi_0), m_3\right)} - \sqrt{r^2 + pr + q} + r\right]. \tag{38}$$

Since $U(\xi) = \int V(\xi) d\xi$ and (8), we get the travelling solutions of Equation (2) from (25), (26), (27), (29), (32), (33), and (38), respectively.

$$u_1(\xi) = \left(-\frac{2}{k_1}\right)^{-1/3} (r_2 + 1)(\xi - \xi_0) - 2\left(-\frac{2}{k_1}\right)^{-2/3} (r_1 - r_2) \cdot \tanh\left(\frac{1}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0)\right), r_1 > r_2,$$

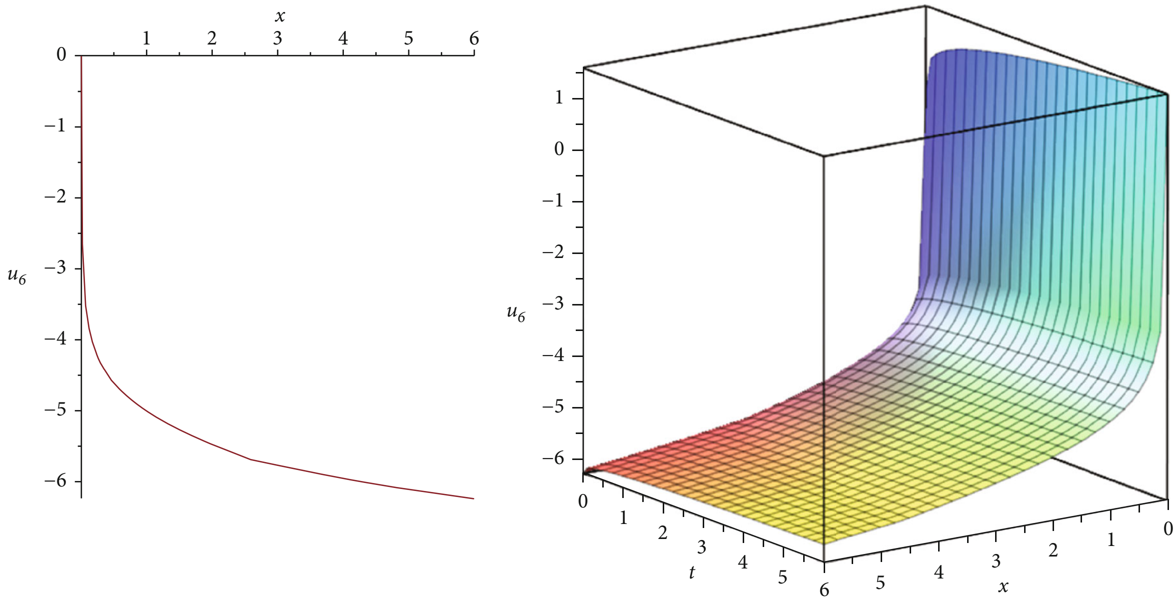
$$u_2(x, y, z, t) = \left(-\frac{2}{k_1}\right)^{-1/3} (r_2 + 1)(\xi - \xi_0) - 2\left(-\frac{2}{k_1}\right)^{-2/3} \cdot (r_1 - r_2) \coth\left(\frac{1}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0)\right), r_1 > r_2,$$

$$u_3(x, y, z, t) = \left(-\frac{2}{k_1}\right)^{-1/3} r_1(\xi - \xi_0) + 2\sqrt{r_2 - r_1}\left(-\frac{2}{k_1}\right)^{-2/3} \cdot \tan\left(\frac{\sqrt{r_2 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0)\right), r_1 < r_2,$$

$$u_4(x, y, z, t) = \left(-\frac{2}{k_1}\right)^{-1/3} r(\xi - \xi_0) - 4\left(-\frac{2}{k_1}\right)^{-2/3} (\xi - \xi_0)^{-1},$$

$$u_5(x, y, z, t) = \left(-\frac{2}{k_1}\right)^{-1/3} \left(r_1 + \frac{r_2 - r_1}{{m_1}^2}\right)(\xi - \xi_0) - \frac{2(r_2 - r_1)}{{m_1}^2\sqrt{r_3 - r_1}}\left(-\frac{2}{k_1}\right)^{-2/3} \cdot E\left(\operatorname{sn}\left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0), m_1\right), m_1\right),$$

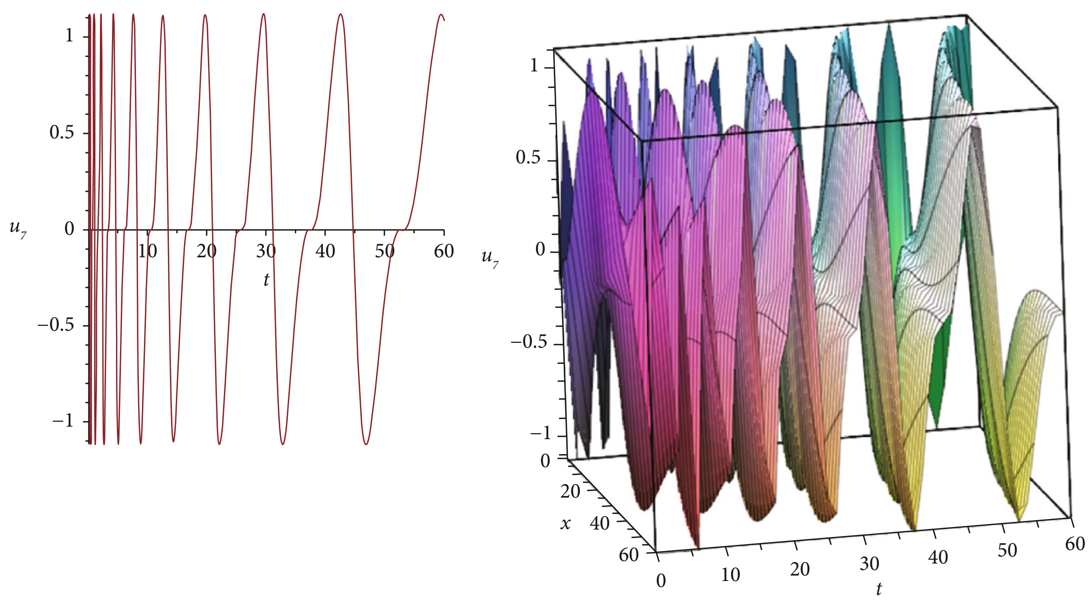
$$u_6(x, y, z, t) = \left(-\frac{2}{k_1}\right)^{-1/3} (r_2 + 1)(\xi - \xi_0) + E\left(\operatorname{sn}\left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0), m_2\right), m_2\right) - \frac{(r_3 - r_2)(-2/k_1)^{-2/3}}{\sqrt{r_3 - r_1}/2({m_2}^2 - 1)} \operatorname{dn} \cdot \left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0), m_2\right) \times sn\left(\frac{\sqrt{r_3 - r_1}}{2}\left(-\frac{2}{k_1}\right)^{1/3} (\xi - \xi_0), m_2\right),$$




(a) $u_6(x,y,z,t)$ when $y=0, z=0$

(b) $u_6(x,y,z,t)$ when $y=1, z=0$

Figure 6: $u_6(x,y,z,t)$ when $k_1=2, k_2=k_3=\nu=\alpha=\beta=\gamma=\eta=1/4, c_0=0, c_1=1, \xi_0=0$.




(a) $u_7(x,y,z,t)$ when $y=1, z=0$

(b) $u_7(x,y,z,t)$ when $z=0, t=1$

Figure 7: $u_7(x,y,z,t)$ when $k_1=2, k_2=1/8, k_3=1, \nu=-24, \alpha=\beta=\gamma=\eta=1/4, c_0=0, c_1=-1, \xi_0=0$.

$$u_7(x,y,z,t)=\left(-\frac{2}{k_1}\right)^{-1/3}\left[(\xi-\xi_0)-E(\mathrm{sn}(M(\xi-\xi_0),m_3),m_3)\right]+\frac{2\mathrm{dn}(M(\xi-\xi_0),m_3)\mathrm{sn}(M(\xi-\xi_0),m_3)}{(1+\mathrm{cn}(M(\xi-\xi_0),m_3))}, \tag{39}$$

where $E(\phi,m)=\int_0^\phi\sqrt{1-m^2\sin^2\phi}\,d\phi$, $M=(-2/k_1)^{-1/3}(r^2+pr+q)^{1/4}$.

## 5. Numerical Simulation

According to the above classification of all single travelling wave solutions to space-time fractional JM equation, we give the corresponding representation of these solutions. By taking concrete parameter values and conditions, we give concrete solutions. This means that all these solutions are realizable. The following contains the 3D and 2D solution graph for the obtained solutions of space-time fractional JM equation. Here, the numerical simulation has been done in Figures 1–7 for showing the nature of the obtained solution. In addition, we also note that (25) and (26) are solitary wave solutions, but $u_1$ and $u_2$ are not solitary wave solutions. Similarly, (32), (33), and (38) are the periodic wave solutions, but $u_5, u_6, u_7$ are the sum of unbounded function and periodic functions. These just illustrate the complexity of JM equation.

## 6. Conclusion

The space-time fractional $(3+1)$-dimensional JM equations is studied by the complete discrimination system method. Compared with the existing literature [33, 35], a series of new exact solutions are obtained, including rational function solutions, Jacobian elliptic function solutions, hyperbolic function solutions, and trigonometric function solutions. It can be seen from the above figures that all solutions can be realized by selecting appropriate parameters, which means that compared with other literatures, we have obtained more abundant traveling wave solutions of $(3+1)$-dimensional Jimbo-Miwa equation with space-time fractional derivative. These solutions may help us to explore new phenomena which appear in Equation (2). This paper gives a new idea to study the dispersive traveling wave solutions of $(3+1$ $)$-dimensional Jimbo-Miwa equation with space-time fractional derivative. If $\alpha, \beta, \gamma$, and $\nu$ take 1, we get the traveling wave solutions to the usual JM Equation (1). Moreover, the complete discrimination system method can also be used to find the exact traveling wave solutions of other coupled systems. In future research work, we will focus on the exact traveling wave solution of more complex coupled systems.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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