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Research Article

New Optical Solitons in Bragg Grating Fibers for the Nonlinear Coupled $(2+1)$-Dimensional Kundu-Mukherjee-Naskar System via Complete Discrimination System Method

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In this paper, the coupled Kundu-Mukherjee-Naskar (KMN) model in Bragg grating fibers is considered to retrieve some new optical soliton solutions in $(2+1)$ dimensions. Plenty of new exact solutions, including rational function solutions and triangle function solutions, in addition to the Jacobian elliptic function solutions, are obtained by using a complete discrimination system method. The 3D-surface plots, 2D-shape plots, and corresponding 2D contour plots of some obtained solutions are drawn, which provides a visualized structures and propagation of solitons. The novel results are new and show the effectiveness of the proposed method.

1. Introduction

The Kundu-Mukherjee-Naskar (KMN) equation was first proposed in 2014 to address rogue waves (RWs) in the ocean [1], which is usually denotes as

$$iq_t + aq_{xy} + ib(qq_x^* - q^*q_x) = 0. \tag{1}$$

In recent years, many mathematicians and physicists have devoted themselves to the research and discussion of this equation. In [2], Qiu et al. considered this model to obtain first-order oceanic rogue wave solution via the Darboux transformation. In [3], Singh et al. obtained higher-dimensional nonlinear wave solutions and study the dynamics of deep water oceanic RWs. In [4], Ekici et al. indicated that the KMN equation is applicable to study the dynamics of soliton propagation through optical fibers in $(2+1)$ dimensions on the bases of the fact that RWs are observed in a crystal fiber [5]. Note that the use of special methods constructing exact solutions of nonlinear differential models (see [6–13]) is a main research area of nonlinear optical science, the optical solitons in KMN equation have been addressed by broad researchers to recover the exact solutions by applying many effective methods including Kudryashov's approach method [14, 15], the tanh function method [16], the new auxiliary equation method [17], the new extended direct algebraic method [18, 19], the method of undetermined coefficients and Lie symmetry [20, 21], the trial equation technique [22], the functional variable method [23], and the modified simple equation approach technique [24]. It is worth mentioning that Yldrm [25] also used the modified simple equation approach technique to discuss a new model of coupled KMN equations in birefringent fibers. Then, Zayed et al. [26] first proposed the coupled KMN model in the Bragg gratings fibers.

In this study, we consider the nonlinear coupled $(2+1)$-dimensional KMN model in the Bragg grating fibers [26]:

$$\begin{aligned} &i\psi_t + a_1\phi_{xy} + i\left[\left(b_1\psi^2 + c_1\phi^2\right)\psi_x^* - \left(d_1|\psi|^2 + e_1|\phi|^2\right)\psi_x\right] \\ &\quad + i\alpha_1\psi_x + \beta_1\phi + \sigma_1\psi^*\phi^2 = 0, \\ &i\phi_t + a_2\psi_{xy} + i\left[\left(b_2\phi^2 + c_2\psi^2\right)\phi_x^* - \left(d_2|\phi|^2 + e_2|\psi|^2\right)\phi_x\right] \\ &\quad + i\alpha_2\phi_x + \beta_2\psi + \sigma_2\phi^*\psi^2 = 0, \end{aligned} \tag{2}$$

where $\psi(x, y, t)$ and $\phi(x, y, t)$ are complex-valued functions that represent the wave profiles, while a_j, b_j, c_j, d_j, e_j, α_j, β_j, and $\sigma_j (j = 1\,; 2)$ are real-valued constants. The parameters a_j $(j = 1\,; 2)$ are the coefficients of dispersion terms. The parameters b_j, c_j, d_j, and e_j are the coefficients of nonlinearity. The parameters α_j, β_j, and σ_j give the intermodal dispersions, the detuning parameters, and the four wave mixing parameters, respectively. In the same work [26], the modified and addendum Kudryashov's method are used to obtain optical solitons for model (2).

Recently, the complete discrimination system for the polynomial method was first proposed by Liu [27] as it is modified form of the discrimination system for high degree polynomial and provides a bit wider range of exact solutions than the famous methods mentioned above. Many authors have solved a lot of models in mathematics, physics, engineering, fluid mechanics, plasma, optical fibers, and other areas of science (see [28–33]). The aim of this paper is to extract the new exact optical solutions of the model (2) after utilizing Liu's method.

The rest of the paper is organized as follows: In Section 2, a glance of discrimination system for polynomial method is given. In Section 3, after taking travelling wave transformation upon (2), the model is reduced to ordinary differential equations (ODEs). Implementing the complete discrimination system method to the ODEs, the exact solutions are obtained. In Section 4, the focus is on the graphical representation for the obtained solutions. We fix parameters to draw the 3D-surface plots, 2D-shape plots, and the corresponding 2D contour plots of some obtained solutions. Finally, the comparison of the obtained results is discussed and conclusions are illustrated in Section 5.

2. Outline of the Complete Discrimination System Method

We consider a nonlinear differential equation takes the form as follows:

$$G(p, p_t, p_x, p_{xt}, p_{xx}, p_{xxx}, \cdots) = 0, \tag{3}$$

where $p = p(t, x)$ is an unknown function and G is a polynomial of all the derivatives of $p = p(t, x)$.

By applying the classical complex traveling wave transformation $p(x, t) = u(\xi)e^{i\phi}$, we can convert Equation (3) into an ordinary differential equation (ODE), which can be written as

$$F\left(u, u', u'', \cdots\right) = 0, \tag{4}$$

where F is a polynomial of u and its derivatives, notation "$'$" denotes the derivative with respect to ξ.

Next, Equation (4) can be reduced to

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}u}{\sqrt{F(u)}}, \tag{5}$$

where ξ_0 is an integral constant and $F(u)$ denotes a n degree polynomial. According to the complete discriminant system method, we can retrieve all types of single wave solutions of Equation (2) from Equation (5) by classifying the roots for polynomial $F(u)$.

3. Applications to the Model

Firstly, we assume Equation (2) have solutions in the form:

$$\psi(x, y, t) = u(\xi)e^{i\zeta(x,y,t)}, \qquad \phi(x, y, t) = v(\xi)e^{i\zeta(x,y,t)},$$
$$\xi = K_1 x + K_2 y - vt, \qquad \zeta(x, y, t) = -k_1 x - k_2 y + \omega t + \theta, \tag{6}$$

where $u(\xi)$ and $v(\xi)$ are the functions stand for the profile for optical pulse, and $\zeta(x, y, t)$ describes the phase portion. Here, the parameters K_1 and K_2 in the amplitude component stand for the direct cosines of the solitons along the x - and y-directions, respectively, and v denotes the soltion velocity. The parameters k_1 and k_2 in the amplitude component signify the frequencies of the solitons along the x- and y -directions, respectively, while ω is the wave number and θ is the phase constant.

Inserting (6) into (2) and decomposing into the real and imaginary parts, we get the real parts for the $u(\xi)$ and $v(\xi)$ as

$$a_1K_1K_2v''(\xi) + (\beta_1 - a_1k_1k_2)v(\xi) + (\alpha_1k_1 - \omega)u(\xi) - (b_1 + d_1)k_1u^3(\xi) + [\sigma_1 - (c_1 + e_1)k_1]u(\xi)v^2(\xi) = 0, \tag{7}$$

$$a_2K_1K_2u''(\xi) + (\beta_2 - a_2k_1k_2)u(\xi) + (\alpha_2k_1 - \omega)v(\xi) - (b_2 + d_2)k_1v^3(\xi) + [\sigma_2 - (c_2 - e_2)k_1]u^2(\xi)v(\xi) = 0, \tag{8}$$

while the imaginary parts are

$$(\alpha_1K_1 - v)u'(\xi) - a_1(k_2K_1 + K_2k_1)v'(\xi) + K_1(b_1 - d_1)u^2(\xi)u'(\xi) + K_1(c_1 - e_1)v^2(\xi)u'(\xi) = 0, \tag{9}$$

$$(\alpha_2K_1 - v)v'(\xi) - a_2(k_1K_2 + K_1k_2)u'(\xi) + K_1(b_2 - d_2)v^2(\xi)v'(\xi) + K_1(c_2 - e_2)u^2(\xi)v'(\xi) = 0. \tag{10}$$

In the view of physical reality, we suppose that

$$v(\xi) = Au(\xi), \tag{11}$$

where $A \neq 0$ is a constant.

Then, implementing the condition (11) in (7)–(8), the real parts are rewritten as

$$a_1K_1K_2Au''(\xi) + [A(\beta_1 - a_1k_1k_2) + (\alpha_1k_1 - \omega)]u(\xi) + \left\{A^2[\sigma_1 - (c_1 + e_1)k_1] - (b_1 + d_1)k_1\right\}u^3(\xi) = 0, \tag{12}$$

$$a_2K_1K_2u''(\xi) + [(\beta_2 - a_2k_1k_2) + A(\alpha_2k_1 - \omega)]u(\xi) + \{A[\sigma_2 - (c_2 + e_2)k_1] - A^3(b_2 + d_2)k_1\}u^3(\xi) = 0. \quad (13)$$

Making Equations (12) and (13) equal for convenient, the coefficients satisfy the following results:

$$a_1A = a_2, \quad (14)$$

$$A(\beta_1 - \alpha_2k_1 + \omega) = \beta_2 - \alpha_1k_1 + \omega, \quad (15)$$

$$A^2[\sigma_1 - (c_1 + e_1)k_1] - (b_1 + d_1)k_1 = A[\sigma_2 - (c_2 + e_2)k_1] - A^3(b_2 + d_2)k_1. \quad (16)$$

Putting (11) into (9) and (10), the imaginary parts become

$$[\alpha_1K_1 - \nu - a_1A(k_2K_1 + K_2k_1)]u'(\xi) + K_1[(b_1 - d_1) + A^2(c_1 - e_1)]u^2(\xi)u'(\xi) = 0, \quad (17)$$

$$[A(\alpha_2K_2 + \nu) - a_2(k_1K_2 + K_1k_2)]u'(\xi) + AK_2[A^2(b_2 - d_2) + (c_2 - e_2)]u^2(\xi)u'(\xi) = 0. \quad (18)$$

Taking into account the linear independence of (17) and (18) and making the coefficients to zero, we obtain the constrain conditions:

$$\alpha_1K_1 - \nu - Aa_1(k_2K_1 + K_2k_1) = 0, \quad A(\alpha_2K_1 - \nu) - a_2(k_1K_2 + K_1k_2) = 0, \quad (19)$$

$$(b_1 - d_1) + A^2(c_1 - e_1) = 0, A^2(b_2 - d_2) + (c_2 - e_2) = 0. \quad (20)$$

Implementing (14) in (19), the velocity of the soliton is obtained as

$$\nu = \alpha_1K_1 - a_2(k_1K_2 + K_1k_2) = \alpha_2K_1 - a_1(k_1K_2 + K_1k_2). \quad (21)$$

Multiplying u' on both sides of (12) and integrating it on ξ, we get

$$\left[u'(\xi)\right]^2 = \frac{(b_1 + d_1)k_1 - A^2[\sigma_1 - (c_1 + e_1)k_1]}{2a_1K_1K_2A}u^4(\xi) + \frac{(\omega - \alpha_1k_1) - A(\beta_1 - a_1k_1k_2)}{a_1K_1K_2A}u^2(\xi) + C_0, \quad (22)$$

where C_0 is an arbitrary constant.

For obtaining exact solutions, the following transformations are selected:

$$u = \pm\sqrt{\left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1K_1K_2A}\right)^{-1/3}w}, \quad \xi_1 = \left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\xi. \quad (23)$$

Inserting (23) into (22), we yield

$$\left[w_{\xi_1}'\right]^2 = w^3(\xi_1) + pw^2(\xi_1) + qw(\xi_1), \quad (24)$$

where q is the an arbitrary constant and coefficient p has the forms:

$$p = 4\frac{(\omega - \alpha_1k_1) - A(\beta_1 - a_1k_1k_2)}{a_1K_1K_2A} \cdot \left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1K_1K_2A}\right)^{-2/3}. \quad (25)$$

It is easy to reformulate Equation (24) with an integral representation as

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{w(w^2 + pw + q)}}, \quad (26)$$

where ξ_0 is the integration constant.

Note that the sign $\Delta = p^2 - 4q$ and p comprise the complete discriminant system for the polynomial $F(w) = w^3 + pw^2 + qw$, and we retrieve the exact solutions of (24) by applying the complete discrimination system method in the paragraph below.

Case 1. $\Delta = 0$. Then, we have $F(w) = w(w + p/2)^2$. When $w > 0$, the exact traveling wave solutions for Equation (2) are recovered as below:

If $-p/2 > 0$, the exact solutions for Equation (2) are

$$\psi_1(x, y, t) = \pm\sqrt{-\frac{p}{2}}\left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x - k_2y + \omega t + \theta)} \times \tanh\left[\frac{\sqrt{-p/2}}{2}\left(\left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x + K_2y - \nu t) - \xi_0\right)\right], \quad (27)$$

$$\phi_1(x,y,t)=\pm A\sqrt{-\frac{p}{2}}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\tanh\cdot\left[\frac{\sqrt{-p/2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right], \tag{28}$$

$$\psi_2(x,y,t)=\pm\sqrt{-\frac{p}{2}}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\coth\cdot\left[\frac{\sqrt{-p/2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right], \tag{29}$$

$$\phi_2(x,y,t)=\pm A\sqrt{-\frac{p}{2}}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\coth\cdot\left[\frac{\sqrt{-p/2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]. \tag{30}$$

If $-p/2<0$, the solution is

$$\psi_3(x,y,t)=\pm\sqrt{\frac{p}{2}}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\tan\cdot\left[\frac{\sqrt{p/2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right], \tag{31}$$

$$\phi_3(x,y,t)=\pm A\sqrt{\frac{p}{2}}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\tan\cdot\left[\frac{\sqrt{p/2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]. \tag{32}$$

If $-p/2=0$, we get

$$\psi_4(x,y,t)=\pm 2\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left[\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right]^{-1},$$

$$\phi_4(x,y,t)=\pm 2A\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left[\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right]^{-1}. \tag{33}$$

Case 2. $\Delta>0$, $q=0$. Then, we have $F(w)=w^2(w+p)$. When $w>-p$, the exact traveling wave solutions for Equation (2) are obtained as follows.

If $0>-p$, we have

$$\psi_5(x,y,t)=\pm\left(-\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left\{p-p\tanh^2\left[\frac{\sqrt{p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]\right\}^{1/2}, \tag{34}$$

$$\phi_5(x,y,t)=\pm A\left(-\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left\{p-p\tanh^2\left[\frac{\sqrt{p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]\right\}^{1/2}, \tag{35}$$

$$\psi_6(x,y,t)=\pm\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left\{p\coth^2\left[\frac{\sqrt{p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]-p\right\}^{1/2}, \tag{36}$$

$$\phi_6(x,y,t)=\pm A\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left\{p\coth^2\left[\frac{\sqrt{p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]-p\right\}^{1/2}. \tag{37}$$

If $0<-p$, we have

$$\psi_7(x,y,t)=\pm\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\sqrt{-p}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\sec\left[\frac{\sqrt{-p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right], \tag{38}$$

$$\phi_7(x,y,t)=\pm A\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\sqrt{-p}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\sec\left[\frac{\sqrt{-p}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right)\right]. \tag{39}$$

Case 3. $\Delta>0$, $q\neq0$, that is, $F(w)=w(w-\lambda_1)(w-\lambda_2)$. We note that, in this case, $\lambda_1\neq\lambda_2\neq0$.

If $0<\lambda_1<\lambda_2$, when $0<w<\lambda_1$, the exact solution for Equation (2) takes the form

$$\psi_8(x,y,t)=\pm\sqrt{\lambda_1}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\operatorname{sn}\left(\frac{\sqrt{\lambda_2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right),m\right),$$

$$\phi_8(x,y,t)=\pm A\sqrt{\lambda_1}\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\operatorname{sn}\left(\frac{\sqrt{\lambda_2}}{2}\left(\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3}\cdot(K_1x+K_2y-\nu t)-\xi_0\right),m\right), \tag{40}$$

when $\lambda_2<w$, the exact solution for Equation (2) takes the form

$$\psi_9(x,y,t)=\pm\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left[\frac{\lambda_2-\lambda_1\operatorname{sn}^2\left(\left(\sqrt{\lambda_2}/2\right)\left(\left((2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t)-\xi_0\right),m\right)}{\operatorname{cn}^2\left(\left(\sqrt{\lambda_2}/2\right)\left(\left((2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t)-\xi_0\right),m\right)}\right]^{1/2},$$

$$\phi_9(x,y,t)=\pm A\left(\frac{2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6}\times e^{i(-k_1x-k_2y+\omega t+\theta)}\times\left[\frac{\lambda_2-\lambda_1\operatorname{sn}^2\left(\left(\sqrt{\lambda_2}/2\right)\left(\left((2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t)-\xi_0\right),m\right)}{\operatorname{cn}^2\left(\left(\sqrt{\lambda_2}/2\right)\left(\left((2(b_1+d_1)k_1-2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t)-\xi_0\right),m\right)}\right]^{1/2}, \tag{41}$$

where $m^2 = \lambda_1/\lambda_2$.

If $\lambda_1 < 0 < \lambda_2$, when $\lambda_1 < w < 0$, the exact solution for Equation (2) takes the form

$$\psi_{10} = \pm\sqrt{-\lambda_1}\left(-\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[-1 + \mathrm{sn}^2\left(\frac{\sqrt{\lambda_2-\lambda_1}}{2}\left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{1/2},$$

$$\phi_{10} = \pm A\sqrt{-\lambda_1}\left(-\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[-1 + \mathrm{sn}^2\left(\frac{\sqrt{\lambda_2-\lambda_1}}{2}\left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{1/2}, \quad (42)$$

when $\lambda_2 < w$, the exact solution for Equation (2) takes the form

$$\psi_{11} = \pm\sqrt{\lambda_2}\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[\mathrm{cn}\left(\frac{\sqrt{\lambda_2-\lambda_1}}{2}\left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{-1},$$

$$\phi_{11} = \pm A\sqrt{-\lambda_2}\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[\mathrm{cn}\left(\frac{\sqrt{\lambda_2-\lambda_1}}{2}\left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{-1}, \quad (43)$$

where $m^2 = -\lambda_1/(\lambda_2 - \lambda_1)$.

If $\lambda_1 < \lambda_2 < 0$, when $\lambda_1 < w < \lambda_2$, the exact solution for Equation (2) takes the form

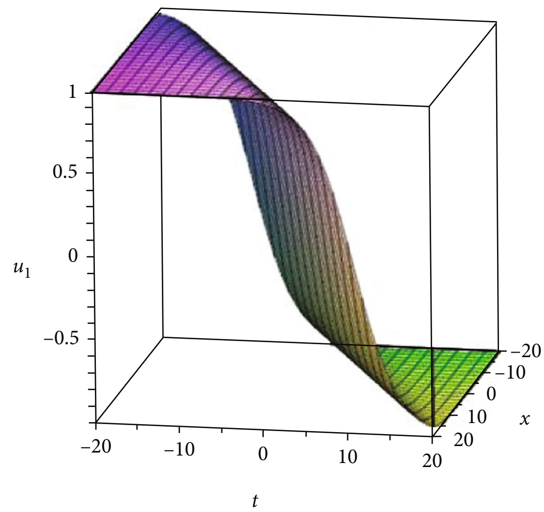
$$\psi_{12} = \pm\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{-a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[\lambda_1 - (\lambda_2-\lambda_1)\mathrm{sn}^2\left(\frac{\sqrt{-\lambda_1}}{2} \cdot \left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{1/2},$$

$$\phi_{12} = \pm A\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{-a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \left[\lambda_1 - (\lambda_2-\lambda_1)\mathrm{sn}^2\left(\frac{\sqrt{-\lambda_1}}{2} \cdot \left(\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{1/3} \cdot (K_1x+K_2y-\nu t) - \xi_0\right), m\right)\right]^{1/2}, \quad (44)$$

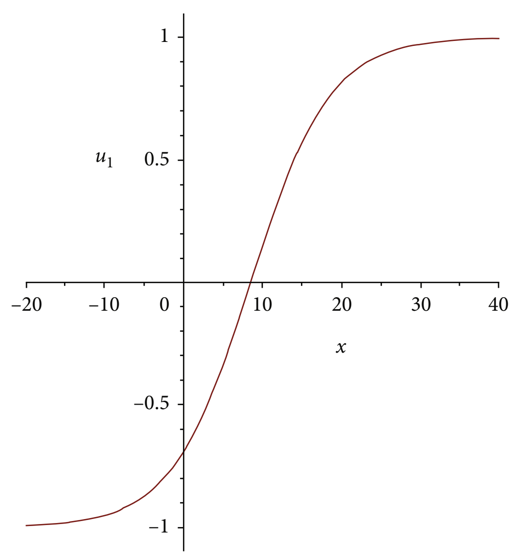
when $0 < w$, the exact solution for Equation (2) takes the form

$$\psi_{13}(x,y,t) = \pm\sqrt{-\lambda_2}\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \frac{\mathrm{sn}\left(\left(\sqrt{-\lambda_1}/2\right)\left(\left((2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t) - \xi_0\right), m\right)}{\mathrm{cn}\left(\left(\sqrt{-\lambda_1}/2\right)\left(\left((2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t) - \xi_0\right), m\right)},$$

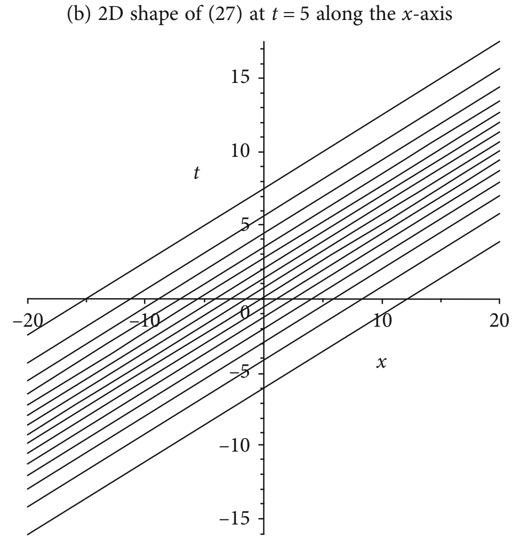
$$\phi_{13}(x,y,t) = \pm A\sqrt{-\lambda_2}\left(\frac{2(b_1+d_1)k_1 - 2A^2[\sigma_1 - (c_1+e_1)k_1]}{a_1K_1K_2A}\right)^{-1/6} \times e^{i(-k_1x-k_2y+\omega t+\theta)} \times \frac{\mathrm{sn}\left(\sqrt{-\lambda_1}/2\left(\left((2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t) - \xi_0\right), m\right)}{\mathrm{cn}\left(\sqrt{-\lambda_1}/2\left(\left((2(b_1+d_1)k_1 - 2A^2[\sigma_1-(c_1+e_1)k_1])/a_1K_1K_2A\right)^{1/3}(K_1x+K_2y-\nu t) - \xi_0\right), m\right)}, \quad (45)$$



(a) 3D surface plot of (27)

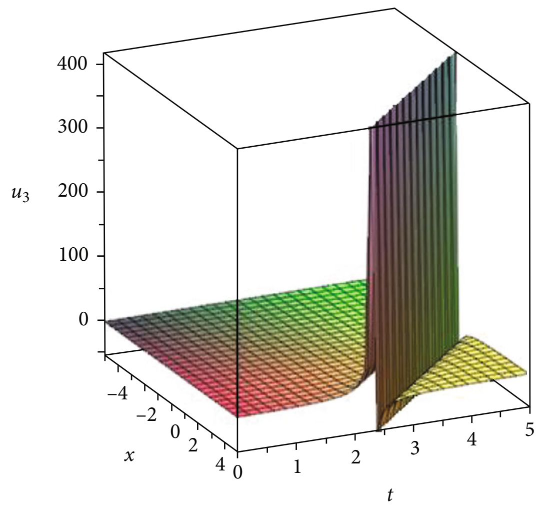


(b) 2D shape of (27) at $t = 5$ along the x-axis

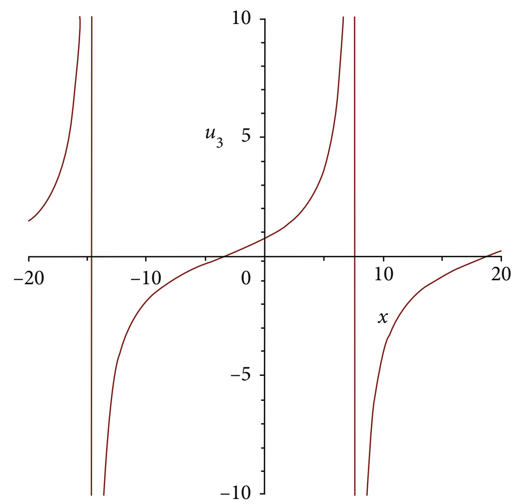


(c) 2D contour plot of u_1

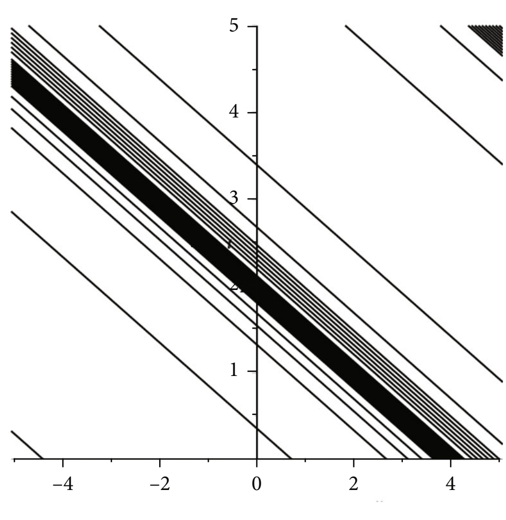
Figure 1: The graph shows solution ψ_1 by assuming $a_1 = 0.1$, $a_2 = 0.2$, $b_1 = 0.24$, $b_2 = 0.03$, $c_1 = 0.03$, $c_2 = 0.06$, $d_1 = 0.16$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $\alpha_1 = 2.08$, $\alpha_2 = 2.04$, $\beta_1 = 0.002$, $\beta_2 = 0.014$, $\sigma_1 = 0.0256$, $\sigma_2 = 0.0512$, $y = 1$, $\theta = 0$, and $\xi_0 = 0$. Letting $K_1 = 0.2$, $K_2 = 0.3$, $k_1 = 0.2$, and $k_2 = 0.1$, then $\nu = 0.4$, $\omega = 0.41$, $p = -2$, and $q = 1$.



(a) 3D surface plot of (31)

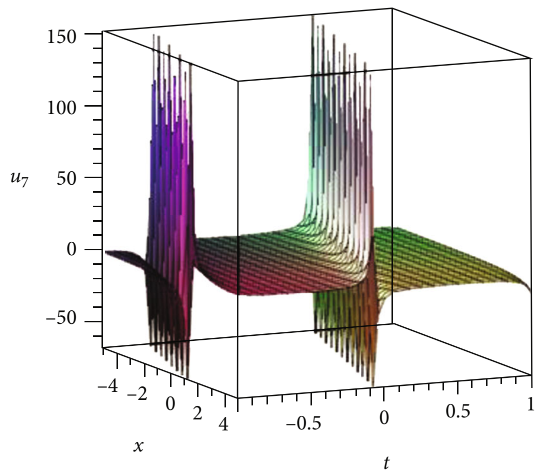


(b) 2D shape of (31) at $t = 1$ along x-axis

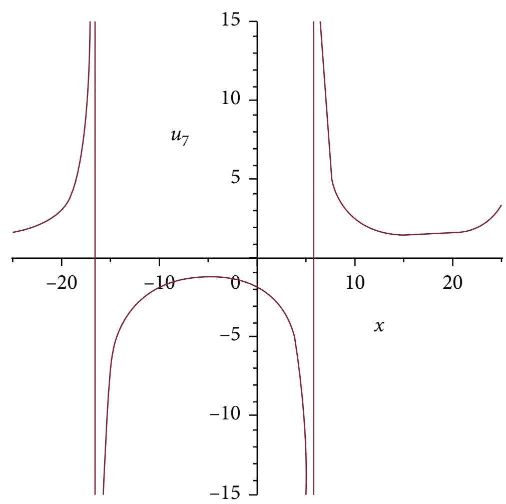


(c) 2D contour plot of u_3

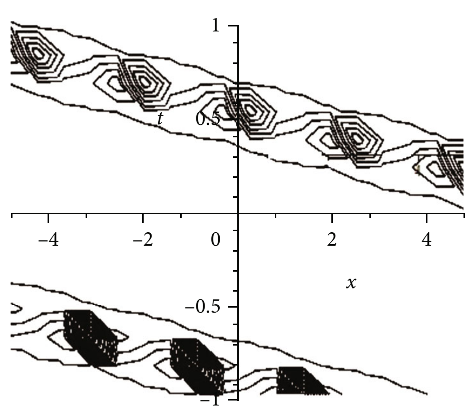
Figure 2: The graph shows solution ψ_3 by assuming $a_1 = 0.1$, $a_2 = 0.2$, $b_1 = 0.24$, $b_2 = 0.03$, $c_1 = 0.03$, $c_2 = 0.06$, $d_1 = 0.16$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $\alpha_1 = 2.08$, $\alpha_2 = 2.04$, $\beta_1 = 0.002$, $\beta_2 = -0.768$, $\sigma_1 = 0.0256$, $\sigma_2 = 0.0512$, $y = 1$, $\theta = 0$, and $\xi_0 = 0$. Letting $K_1 = 0.2$, $K_2 = 0.3$, $k_1 = 0.2$, and $k_2 = 0.1$, then $\nu = 0.4$, $\omega = -0.372$, $p = 4$, and $q = 1$.



(a) 3D surface plot of (38)

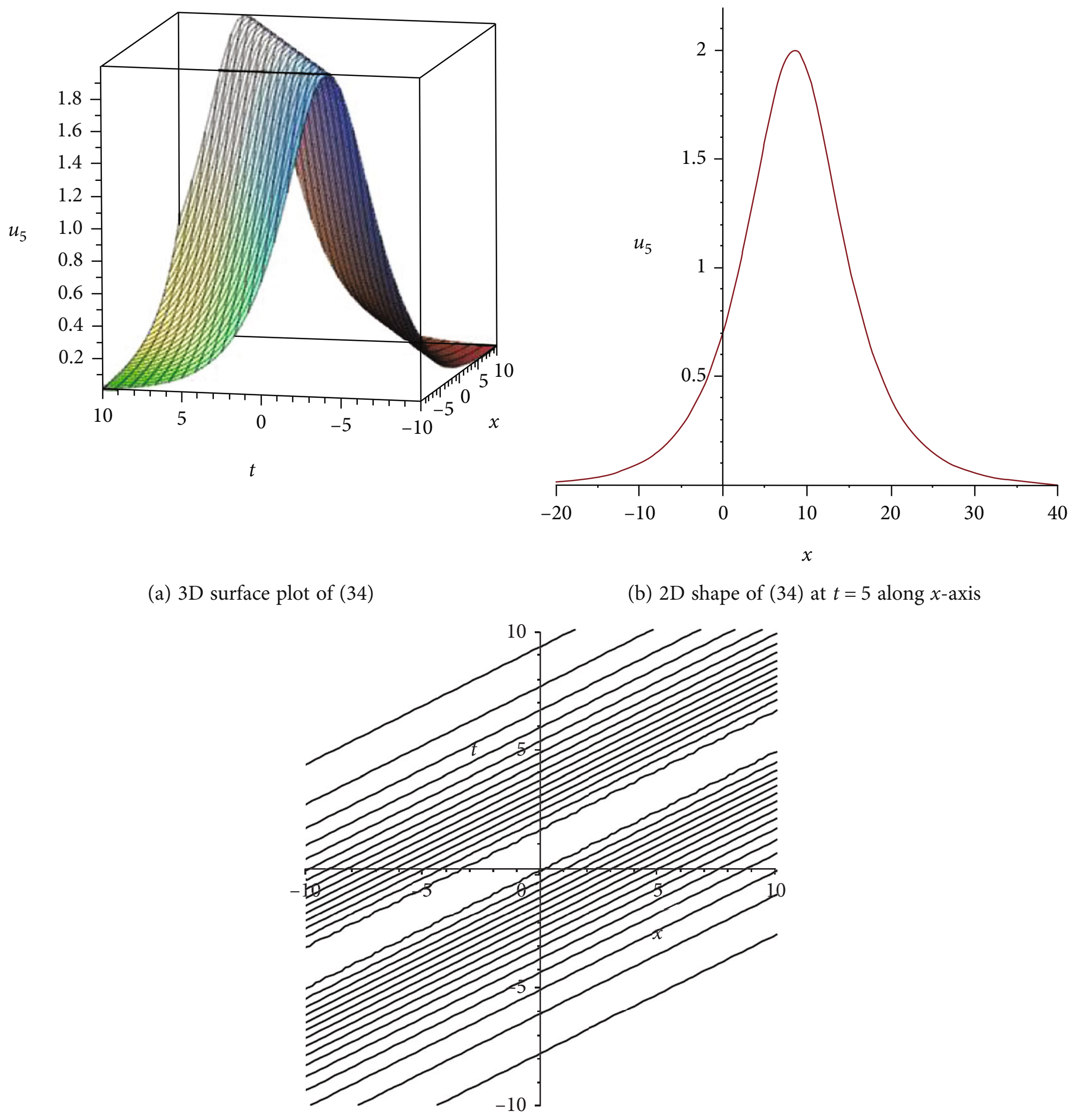


(b) 2D shape of (38) at $t = 1$ along x-axis



(c) 2D contour plot of u_7

FIGURE 3: The graph shows solution ψ_7 by assuming $a_1 = 0.1$, $a_2 = 0.2$, $b_1 = 0.24$, $b_2 = 0.03$, $c_1 = 0.03$, $c_2 = 0.06$, $d_1 = 0.16$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $\alpha_1 = -1.92$, $\alpha_2 = -1.96$, $\beta_1 = 0.002$, $\beta_2 = 0.006$, $\sigma_1 = 0.0256$, $\sigma_2 = 0.0512$, $y = 3$, $\theta = 0$, and $\xi_0 = 0$. Letting $K_1 = 0.2$, $K_2 = 0.3$, $k_1 = 0.2$, and $k_2 = 0.1$, then $\nu = -0.4$, $\omega = -0.39$, $p = -2$, and $q = 0$.



(a) 3D surface plot of (34)

(b) 2D shape of (34) at $t = 5$ along x-axis

(c) 2D contour plot of u_5

FIGURE 4: The graph shows solution ψ_5 by assuming $a_1 = 0.1$, $a_2 = 0.2$, $b_1 = 0.24$, $b_2 = 0.03$, $c_1 = 0.03$, $c_2 = 0.06$, $d_1 = 0.16$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $\alpha_1 = -1.92$, $\alpha_2 = -1.96$, $\beta_1 = 0.002$, $\beta_2 = 0.032$, $\sigma_1 = 0.0295$, $\sigma_2 = 0.059$, $y = 1$, $\theta = 0$, and $\xi_0 = 0$. Letting $K_1 = 0.2$, $K_2 = 0.3$, $k_1 = 0.2$, and $k_2 = 0.1$, then $\nu = -0.4$, $\omega = -0.372$, $p = 4$, and $q = 0$.

TABLE 1: Comparison between Zayed et al. ([26]) solutions and our solutions.

Zayed et al. (2022) solutions	Our solutions
(i) If $L_1 = 1/2$, $L_2 = -1$, $\varepsilon = 1$ then solutions	(i) If $q = 1$, $p = -2$, then we obtain
(35) reduced to $u(\xi) = -1/\sqrt{2}\tanh(1/2(\xi - \xi_0))$	$u_1(\xi) = \pm \frac{1}{\sqrt{2}} \tanh\left(\frac{1}{2}(\xi - \xi_0)\right)$
(ii) If $L_1 = 1/2$, $L_2 = -1$, $\varepsilon = -1$ then solutions	(ii) If $q = 1$, $p = -2$, then we obtain
(37) Reduced to $u(\xi) = -1/\sqrt{2}\coth(1/2(\xi - \xi_0))$	$u_2(\xi) = \pm \frac{1}{\sqrt{2}} \coth\left(\frac{1}{2}(\xi - \xi_0)\right)$

where $m^2 = (\lambda_2 - \lambda_1)/-\lambda_1$.

Case 4. $\Delta < 0$. When $w > 0$, we obtain the exact traveling wave solutions for Equation (2) as

$$\psi_{14}(x, y, t)\psi_{14}(x, y, t) = \pm\sqrt[4]{q} \times \left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1 K_1 K_2 A}\right)^{-1/6} \times e^{i(-k_1 x - k_2 y + \omega t + \theta)} \times \left[\frac{1 - \mathrm{cn}\left(\sqrt[4]{q}\left(\left((2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1])/a_1 K_1 K_2 A\right)^{1/3}(K_1 x + K_2 y - \nu t) - \xi_0\right), m\right)}{1 + \mathrm{cn}\left(\sqrt[4]{q}\left(\left((2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1])/a_1 K_1 K_2 A\right)^{1/3}(K_1 x + K_2 y - \nu t) - \xi_0\right), m\right)}\right]^{1/2},$$

$$\phi_{14}(x, y, t) = \pm A\sqrt[4]{q} \times \left(\frac{2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1]}{a_1 K_1 K_2 A}\right)^{-1/6} \times e^{i(-k_1 x - k_2 y + \omega t + \theta)} \times \left[\frac{1 - \mathrm{cn}\left(\sqrt[4]{q}\left(\left((2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1])/a_1 K_1 K_2 A\right)^{1/3}(K_1 x + K_2 y - \nu t) - \xi_0\right), m\right)}{1 + \mathrm{cn}\left(\sqrt[4]{q}\left(\left((2(b_1 + d_1)k_1 - 2A^2[\sigma_1 - (c_1 + e_1)k_1])/a_1 K_1 K_2 A\right)^{1/3}(K_1 x + K_2 y - \nu t) - \xi_0\right), m\right)}\right]^{1/2}, \tag{46}$$

where $m^2 = (2\sqrt{q} - p)/4\sqrt{q}$.

4. Graphical Representation

In this section, we graphically illustrate the amplitude profile of some retrieved solutions of four types by using Maple software. Graphs 1-4 show the 2D, 3D, and corresponding contour plots for the solutions ψ_1, ψ_3, ψ_5, and ψ_7, respectively, by taking suitable choice of free parameters satisfying all the constrict conditions (14)–(16) and (19)–(20). Figure 1 shows the 2D and 3D views and contour plot of kink soliton solution of (27) by taking suitable parameters. Figures 2 and 3 elaborate both cases of singular periodic solutions of (31) and (38) in 2D and 3D shapes and contour plot. The 2D and 3D views and contour plot of the bright soliton solution of (34) is depicted in Figure 4. Figures 1–4 express an insight view of the structures for the solutions of (2) which give us a better understanding of the propagation and mechanism for the origin system.

5. Conclusion

The governing model in Bragg grating fibers for the coupled Kundu-Mukherjee-Naskar model was examined on purpose of obtaining new optical soliton solutions. The complete discrimination system for the polynomial method was adopted to uncover the rational function solutions, triangle function solutions, and the Jacobian elliptic function solutions of this model. All of the derived solutions are guaranteed by putting them back into the original equation. Many other structures can be obtained by choosing free parameters. Comparing the results with the works done in [26], some of our obtained solutions are in good agreement with the published results which is presented in Table 1. The solutions (31) and (32) and (38) and (39) were not obtained by Zayed et al. [26]. The listed solutions in Case 3 and 4 are reported for the first time and can hardly be obtained by other methods. As a rather newly proposed system, there is a lot of scope to extend the horizon in this context. For this reason, additional techniques such as the Lie group, the new extended direct algebraic method, and the new auxiliary equation method will be considered on this model. These details shall be revealed, respectively, with time.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Chen Peng provided resources; did the investigation, methodology, writing—review and editing, and acquired the software. Furong Zhang did the validation. Hongwei Zhao acquired the software and did the data curation and formal analysis. Zhao Li did the project administration, supervision, and methodology. All authors read and agreed to the published version of the manuscript.

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References

[1] A. Kundu, A. Mukherjee, and T. Naskar, "Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents," *Proceedings of the Royal Society A:*

Mathematical, Physical and Engineering Sciences, vol. 470, no. 2164, article 20130576, 2014.

[2] D. Qiu, Y. Zhang, and J. He, "The rogue wave solutions of a new (2+1)-dimensional equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 30, no. 1-3, pp. 307–315, 2016.

[3] S. Singh, A. Mukherjee, K. Sakkaravarthi, and K. Murugesan, "Higher dimensional localized and periodic wave dynamics in an integrable (2+1)-dimensional deep water oceanic wave model," *Waves in Random and Complex Media*, pp. 1–20, 2021.

[4] M. Ekici, A. Sonmezoglu, A. Biswas, and M. R. Belic, "Optical solitons in (2+1) dimensions with Kundu-Mukherjee-Naskar equation by extended trial function scheme," *Chinese Journal of Physics*, vol. 57, pp. 72–77, 2019.

[5] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," *Nature*, vol. 450, no. 7172, pp. 1054–1057, 2007.

[6] H.-O. Roshid, M. R. Kabir, R. C. Bhowmik, and B. K. Datta, "Investigation of solitary wave solutions for Vakhnenko-Parkes equation via exp-function and $\mathrm{Exp}(-\phi(\xi))$-expansion method," *Springer Plus*, vol. 3, p. 692, 2014.

[7] H. O. Roshid, M. A. Akbar, M. N. Alam, M. F. Hoque, and N. Rahman, "New extended (G'/G)-expansion method to solve nonlinear evolution equation: the (3+1)-dimensional potential-YTSF equation," *Springer Plus*, vol. 3, p. 122, 2014.

[8] M. S. Ullah, M. Z. A. Harun-Or-Roshid, A. Biswas et al., "Optical soliton polarization with Lakshmanan-Porsezian-Daniel model by unified approach," *Results in Physics*, vol. 22, article 103958, 2021.

[9] M. Zillur Rahman, Z. Ali, and H.-O. Roshid, "Closed form soliton solutions of three nonlinear fractional models through proposed improved Kudryashov method," *Chinese Physics B*, vol. 30, no. 5, article 050202, 2021.

[10] M. F. Hoque, "Optical soliton solutions of the Biswas-Arshed model by the tan $(\ominus/2)$ expansion approach," *Physica Scripta*, vol. 95, no. 7, article 075219, 2020.

[11] S. El-Ganaini and M. O. Al-Amr, "New abundant solitary wave structures for a variety of some nonlinear models of surface wave propagation with their geometric interpretations," *Mathematical Methods in the Applied Sciences*, vol. 1, p. 27, 2022.

[12] O. Mohammed, "N1-soliton solution for Schrödinger equation with competing weakly nonlocal and parabolic law nonlinearities," *Communications in Theoretical Physics*, vol. 72, no. 6, article 065503, 2020.

[13] N. M. Rasheed, M. O. Al-Amr, E. A. Az-Zo'bi, M. A. Tashtoush, and L. Akinyemi, "Stable optical solitons for the higher-order non-Kerr NLSE via the modified simple equation method," *Mathematics*, vol. 9, no. 16, p. 1986, 2021.

[14] N. A. Kudryashov, "General solution of traveling wave reduction for the Kundu-Mukherjee-Naskar model," *Optik*, vol. 186, pp. 22–27, 2019.

[15] I. Onder, A. Secer, M. Ozisik, and M. Bayram, "On the optical soliton solutions of Kundu-Mukherjee-Naskar equation via two different analytical methods," *Optik*, vol. 257, article 168761, 2022.

[16] H. M. Baskonus, J. Guirao, A. Kumar, F. Causanilles, and G. R. Bermudez, "Instability modulation properties of the (2 + 1)-dimensional Kundu-Mukherjee-Naskar model in travelling wave solutions," *Modern Physics Letters B*, vol. 35, no. 13, p. 2150217, 2021.

[17] D. Kumar, G. C. Paul, T. Biswas et al., "Optical solutions to the Kundu-Mukherjee-Naskar equation: mathematical and graphical analysis with oblique wave propagation," *Physica Scripta*, vol. 96, no. 2, article 025218, 2021.

[18] H. Günerhan, F. S. Khodadad, H. Rezazadehy, and M. M. A. Khater, "Exact optical solutions of the (2+1) dimensions Kundu-Mukherjee-Naskar model via the new extended direct algebraic method," *Modern Physics Letters B*, vol. 34, no. 22, article 2050225, 2020.

[19] A. Jhangeer, A. R. Seadawy, F. Ali, and A. Ahmed, "New complex waves of perturbed Shrodinger equation with Kerr law nonlinearity and Kundu-Mukherjee-Naskar equation," *Results in Physics*, vol. 16, article 102816, 2020.

[20] A. Biswas, J. Vega-Guzman, A. Bansal et al., "Optical dromions, domain walls and conservation laws with Kundu-Mukherjee- Naskar equation via traveling waves and Lie symmetry," *Results in Physics*, vol. 16, article 102850, 2020.

[21] R. Cimpoiasu, H. Rezazadeh, D. A. Florian, H. Ahmad, K. Nonlaopon, and M. Altanji, "Symmetry reductions and invariant-group solutions for a two-dimensional Kundu-Mukherjee-Naskar model," *Results in Physics*, vol. 28, article 104583, 2021.

[22] Y. Yıldırım, "Optical solitons to Kundu-Mukherjee-Naskar model with trial equation approach," *Optik*, vol. 183, pp. 1061–1065, 2019.

[23] H. Rezazadeh, A. Kurt, A. Tozar, O. Tasbozan, and S. M. Mirhosseini-Alizamini, "Wave behaviors of Kundu-Mukherjee-Naskar model arising in optical fiber communication systems with complex structure," *Optical and Quantum Electronics*, vol. 53, no. 6, p. 317, 2021.

[24] Y. Yıldırım, "Optical solitons to Kundu-Mukherjee-Naskar model with modified simple equation approach," *Optik*, vol. 184, pp. 247–252, 2019.

[25] Y. Yıldırım, "Optical solitons to Kundu-Mukherjee-Naskar model in birefringent fibers with modified simple equation approach," *Optik*, vol. 184, pp. 121–127, 2019.

[26] E. M. Zayed, R. Shohib, and M. E. Alngar, "Optical solitons in Bragg gratings fibers for the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation using two integration schemes," *Optical and Quantum Electronics*, vol. 54, no. 1, article 16, 2022.

[27] C. S. Liu, "Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations," *Computer Physics Communications*, vol. 181, no. 2, pp. 317–324, 2010.

[28] Y. Kai, "The classification of the single travelling wave solutions to the variant Boussinesq equations," *Pramana*, vol. 87, no. 4, pp. 59–63, 2016.

[29] C. S. Liu, "Classification of all single travelling wave solutions to Calogero-Degasperis-Focas equation," *Communications in Theoretical Physics*, vol. 48, no. 4, pp. 601–604, 2007.

[30] Y. Y. Xie, Z. Y. Yang, and L. F. Li, "New exact solutions to the high dispersive cubic-quintic nonlinear Schrodinger equation," *Physics Letters A*, vol. 382, no. 36, pp. 2506–2514, 2018.

[31] Z. Li, T. Y. Han, and C. Huang, "Exact single traveling wave solutions for generalized fractional Gardner equations," *Mathematical Problems in Engineering*, vol. 2020, no. 2020, Article ID 8842496, 2020.

[32] Z. Li and T. Y. Han, "Classification of all single traveling wave solutions of fractional perturbed Gerdjikov–Ivanov equation," *Mathematical Problems in Engineering*, vol. 2021, no. 2021, Article ID 1283083, 2021.

[33] C. Peng, Z. Li, and H. W. Zhao, "New exact solutions to the Lakshmanan-Porsezian-Daniel equation with Kerr law of non-linearity," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7340373, 2022.