



## **Numerical Modeling of Biological Oxygen Demand (BOD<sub>5</sub>) Load in Apa Mmini Stream**

**Ogbebor Daniel<sup>1\*</sup>, Nwaogazie L. Ify<sup>1</sup>, Momoh O. L. Yusuf<sup>1</sup>  
and Ndekwu B. Onyedikachukwu<sup>2</sup>**

<sup>1</sup>Department of Civil and Environmental Engineering, University of Port Harcourt, River State, Nigeria.

<sup>2</sup>Department of Civil Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria.

### **Authors' contributions**

*This work was carried out in collaboration among all authors. Authors MOLY and NLI designed the study and wrote the protocol. Author OD managed the analyses of the study. Author NBO wrote the first draft of the manuscript performed the statistical analysis and managed the literature searches. All authors read and approved the final manuscript.*

### **Article Information**

DOI: 10.9734/JERR/2020/v12i317081

#### **Editor(s):**

(1) Dr. Djordje Cica, University of Banja Luka, Bosnia and Herzegovina.

#### **Reviewers:**

(1) Ali Fadhil Naser, Al-Furat Al-Awsat Technical University, Iraq.

(2) Asad Ullah, Kohat University of Science & Technology, Pakistan.

(3) Aiwen Wang, Beijing Information Science and Technology University, China.

Complete Peer review History: <http://www.sdiarticle4.com/review-history/54653>

**Original Research Article**

**Received 12 February 2020**

**Accepted 19 April 2020**

**Published 09 May 2020**

### **ABSTRACT**

**Aim:** The study aimed at utilizing finite element modeling approach for extrapolation of BOD<sub>5</sub> along the longitudinal length of the stream.

**Study Design:** The linear element method was used for analyzing the Biological Oxygen Demand (BOD) transport along the longitudinal length of the stream using the Advective dispersive equation (ADE) as the governing equation. In this study, it was assumed that dispersion was negligible.

**Methodology:** The BOD<sub>5</sub> transport within the stream was modeled using finite element approach. The equation for the stream water pollution modeling was used and the descriptive variables are described as; D is the dispersion coefficient, C is the unknown or state variable (concentration of pollutant or dissolved salt), V is the velocity or convection parameter, k is the applied source or sink and t is the time coordinates.

**Results:** Prediction results of the finite element model showed that BOD<sub>5</sub> decreased downstream from the point of slaughterhouse effluent expulsion. It was observed that both the observed and predicted values of BOD<sub>5</sub> correlated strongly with a correlation coefficient of 0.9686. The decrease

\*Corresponding author: Email: danogbe3k@yahoo.com;

in BOD<sub>5</sub> downstream proves that there is a low rate of re-aeration of the stream; hence the effluent still exceeds the FMEnv recommended standard of 50 mg/l as it travels downstream along the stream profile. The prediction discloses that the river is highly polluted for long stretch of the watercourse, thus implying that selected system of treatment method must be put in place.

**Conclusion:** Finite element modeling approach was utilized for the extrapolation of BOD<sub>5</sub> in Apa Mmini stream. The outcomes of the simulation showed that the Apa Mmini stream is highly polluted with BOD<sub>5</sub> of 111.8 mg/l which is far above the FMEnv recommended standard of 50 mg/l.

*Keywords: Finite element modeling; Biochemical Oxygen Demand (BOD); linear shape function; energy requirement.*

## 1. INTRODUCTION

A major utmost polluted resources on earth as an upshot of anthropogenic activities; hence most readily available is surface water. The outpour of unprocessed wastewater into shallow water bodies such as streams, rivers, ponds and oceans produces the predisposition of contamination of such water environments. This contamination of shallow water bodies, triggered through the activities of humans, is an increasing occurrence globally [1]. In Nigeria, surface water contamination is being concomitant with shallow runoff, industrial emission, cold-room discharge, domestic waste and slaughterhouse effluent. However, emphasis would be placed on effluent produced from slaughterhouse processes.

The slaughterhouse industry is a significant component of the livestock industry providing domestic meat supply and jobs for over 170 million people in Nigeria [2]. In Nigeria, slaughterhouses are situated indiscriminately without facilities for appropriate supervision of the wastewater discharge, which are ultimately discharged into streams, rivers and water bodies that still serve as key sources of household water for many communities [3]. As a result, human lives have been compromised due to consumption of heavily contaminated water high in fecal coliform, total solids, nitrates and other physico-chemical parameters.

Results from several researches have made known that direct discharge of wastewater carrying animal wastes into the streams increase pollution at the discharge point and depress oxygen concentration further downstream. Diesch [4] reported increase in biochemical oxygen demand when river water near 200-cow dairy feedlot was sampled 60 m below the intersection of discharge of effluent when compared with upstream samples. Nevertheless, water quality was not evaluated further downstream within the intersection of discharge.

Little is known of the extent to which downstream animal wastes in the run-off pollute waters for trivial to moderate magnitude dairies [5].

The majority of the slaughterhouses in Nigeria are situated in close proximity to streams, natural ponds or rivers and a typical example is the Elenwo slaughterhouse that discharges its effluent into Apa Mmini stream. As an upshot, a high likelihood of civic wellbeing and environmental concerns could result from this practice. The slaughterhouse produces several gallons of blood, intestinal contents and tissues introduced into the Apa Mmini stream daily.

Elenwo slaughterhouse effluent is released into Apa Mmini stream exclusively without any practice of treatment which impacts negatively on the water quality evaluation of the stream, nutrient enrichment, loss of dissolved oxygen, proliferation of pathogenic microorganisms and other unsightly conditions. The river is highly coloured and turbid with visible suspended particles despite the fact that the stream still serves as a source of household water hoard to the community. Hence, the need to estimate the concentration of BOD<sub>5</sub> along the longitudinal section of the Apa Mmini stream using the finite element numerical modeling method.

Finite element as a numerical method used for solving problems for which there is no analytical solution or for which the analytical solution is hard to come by is a mathematical tool used in the derivation of models to solving physical parameters [6]. It is used in solving several engineering problems. Since no chemical reaction is assumed to take place during sludge filtration, finite element can be used in deriving equations to describe the system.

Finite element starts by discretizing the region of interest into a finite number of elements. The

nodal points of the elements allow for writing a shape or distribution function. Polynomials are the most applied interpolation functions in finite element approximation. The element equations are defined using the distribution function and when the element equations are combined, they yield a continuous equation that can approximate the system solution. The nodal points and corresponding functional values with shape function are used to write the finite element approximation [7]. With the current computational methods and resources available, it is not clear whether or not using the FEM or modified FDM will provide an advantage over the other. However, in the early days of numerical analysis, one of the major advantages of using the finite element method was the simplicity and ease that FEM allows to solve complex and irregular two dimensional problems [8,9, 10].

$$-V \frac{\partial c}{\partial t} + D \frac{\partial^2 c}{\partial x^2} - kc = \frac{\partial c}{\partial t} \quad (\text{source: [7]}) \quad (2.1)$$

Where,

D is the dispersion coefficient, C is the unknown or state variable (concentration of pollutant or dissolved salt), V is the velocity or convection parameter, k is the applied source or sink and t is the time coordinates [7].

$$\frac{\partial c}{\partial t} = -V \frac{\partial c}{\partial x} + D \frac{\partial^2 c}{\partial x^2} - kc \quad (2.2)$$

Where,

D is the dispersion coefficient, C is the unknown or state variable (concentration of pollutant or dissolved salt), V is the velocity or convection parameter, k is the applied source or sink and t is the time coordinates.

Where at steady state, we have

$$\frac{\partial c}{\partial t} = 0 \quad (2.3)$$

Assuming, dispersion coefficient being negligible;

$$0 = -V \frac{\partial c}{\partial x} - kc \quad (2.4)$$

Using the Galerkins weighted residual method of finite element;

$$\int_0^T NT \left( -V \frac{\partial c}{\partial x} - kc \right) dt = 0 \quad (2.5)$$

$$-V \int_0^T NT \frac{\partial c}{\partial x} dt - \int_0^T NTkc dt = 0 \quad (2.6)$$

## 2. METHODOLOGY

Considering the case of Apa Mmini stream which receives BOD load from Elemenwo Slaughterhouse that result in deteriorating quality of the stream, the BOD<sub>5</sub> transport within the stream was modeled using finite element approach. The linear element method was used for analyzing the BOD transport along the longitudinal length of the stream using the Advective dispersive equation (ADE) as the governing equation. In this study, it was assumed that dispersion was negligible. The equation for the stream water pollution modeling is given as:

Assembling element equation into global equations, we assumed a one-dimension stretch, divided into three elements with four nodes (Linear option) as shown in Fig. 1.

On the linear basis function

$$c = [N]\{c_i\} = \begin{bmatrix} \left(1 - \frac{t}{T}\right) \frac{t}{T} \\ \frac{t}{T} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} \quad (2.7)$$

And

$$\begin{aligned} & \frac{\partial c}{\partial t} \\ &= \frac{1}{T} [-1 \ 1] \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} \end{aligned} \quad (2.8)$$

Where,

C is a continuous variable and  $c_i$  is a piece wise continuous equivalent. On substitution to various terms we have;

**Term 1**

$$-\int_0^T N^T v \frac{\partial c}{\partial t} dt = -v \int_0^T N^T \frac{\partial c}{\partial t} dt \quad (2.9)$$

$$\begin{aligned} &= -v \int_0^T \begin{bmatrix} 1 - \frac{t}{T} \\ \frac{t}{T} \end{bmatrix} \frac{1}{T} [-1 \ 1] \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} \\ &= -v \int_0^T \frac{1}{T} \begin{bmatrix} -\left(1 - \frac{t}{T}\right)\left(1 - \frac{t}{T}\right) \\ -\frac{t}{T} \quad \frac{t}{T} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} dt \end{aligned} \quad (2.10)$$

$$= \frac{-v}{T} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad (2.11)$$

**Term 2**

$$\int_0^T N^T - kcdt \quad (2.12)$$

$$\begin{aligned} &= -k \int_0^T N^T c dt \\ &= -k \int_0^T \begin{bmatrix} 1 - \frac{t}{T} \\ \frac{t}{T} \end{bmatrix} \begin{bmatrix} 1 - \frac{t}{T} & \frac{t}{T} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} dt \\ &= -k \int_0^T \begin{bmatrix} \left(1 - \frac{2t}{T} + \frac{t^2}{T^2}\right) \left(\frac{t}{T} - \frac{t^2}{T^2}\right) \\ \left(\frac{t}{T} - \frac{t^2}{T^2}\right) \frac{t^2}{T^2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} dt \\ &= \frac{-Tk}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \end{aligned} \quad (2.13)$$

Combing term 1 and term 2, we have, the element equation as;

$$= \frac{-v}{T} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{Tk}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} D_1 + 2D_2 & -D_1 + D_2 \\ -D_1 + D_2 & D_1 + 2D_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0 \quad (2.14)$$

$$[D]\{C\} = \{O\}.$$

Where;

$$D_1 = \frac{-v}{T}, \quad D_2 = \frac{-Tk}{6}$$

Hence  $T = \text{Time step}$  (but  $t$  is anytime)

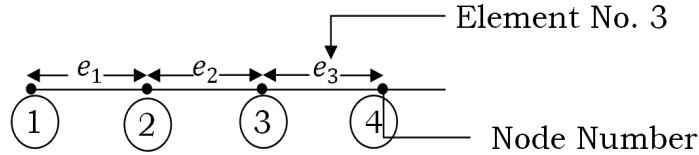


Fig. 1. Linear option sketch

The element assemblage for linear option results is as follows

$$\begin{bmatrix} D_1 + 2D_2 & -D_1 + D_2 & 0 & 0 \\ -D_1 + D_2 & 2D_1 + 4D_2 & -D_1 + D_2 & 0 \\ 0 & -D_1 + D_2 & 2D_1 + 4D_2 & -D_1 + D_2 \\ 0 & 0 & -D_1 + D_2 & D_1 + 2D_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.15)$$

### 3. RESULTS AND DISCUSSION

#### 3.1 Finite Element Prediction of Downstream BOD<sub>5</sub>

The usage of Galerkin's residual method has shown the possibility of applying finite element to

predict the stream BOD<sub>5</sub> profile. Using the Advective dispersive equation (ADE) on a linear shape function bases, where dispersion was assumed negligible, the following element equation was generated (Equation 3.1).

$$\frac{v}{T} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{Tk}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0 \quad (3.1)$$

Upon grouping of constituent equation the Equation 3.2 was obtained;

$$\begin{bmatrix} D_1 + 2D_2 & -D_1 + D_2 & 0 & 0 \\ -D_1 + D_2 & 2D_1 + 4D_2 & -D_1 + D_2 & 0 \\ 0 & -D_1 + D_2 & 2D_1 + 4D_2 & -D_1 + D_2 \\ 0 & 0 & -D_1 + D_2 & D_1 + 2D_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.2)$$

Considering the boundary conditions as;

$$\text{BOD}_5 = 408.4 \text{ mg/l at } x=0$$

$$\text{BOD}_5 = 111.8 \text{ mg/l at } x=L$$

Then the resulting reduced matrix is as follows;

$$\begin{bmatrix} 1.28012 & -1.279942 & & & \\ -1.279942 & 2.56023 & -1.279942 & & \\ & -1.279942 & 2.56023 & -1.279942 & \\ & & -1.279942 & 1.28012 & \\ & & & & \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Upon solving these matrixes, the value of BOD<sub>5</sub> obtained at various locations (S1, S2, S3 and S4) of Apa Mmini Stream were 408.4, 332.2, 181.2 and 111.8 respectively. This was very aligned to the observed field values of 408.4, 309.46, 210.6 and 111.8. A display of the experimental and projected BOD<sub>5</sub> concentration values is as obtainable in Fig. 2.

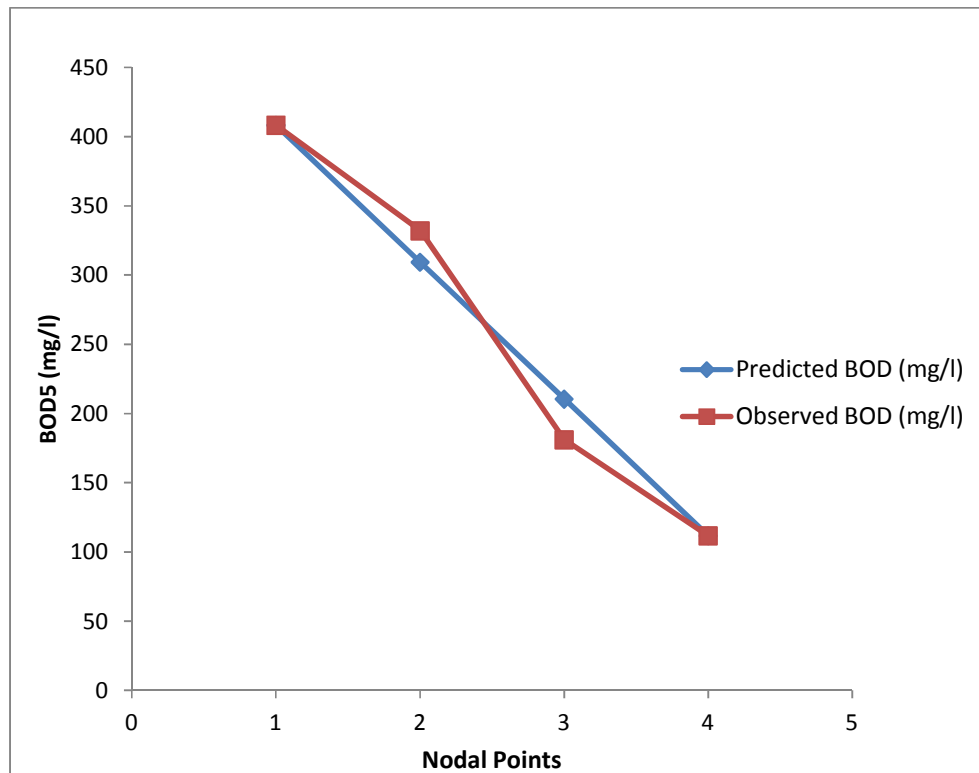


Fig. 2. Plot of observed versus predicted BOD<sub>5</sub> Concentration along the longitudinal length of the Apa Mmini Stream

### 3.2 Finite Element Prediction of Downstream BOD<sub>5</sub>

Prediction results of the finite element model showed that BOD<sub>5</sub> decreased downstream from the point of slaughterhouse effluent expulsion. This decrease downstream proves that there is a low rate of re-aeration of the stream; hence the effluent still exceeds the FMEnv recommended standard of 50mg/l as it travels downstream along the stream profile. The prediction discloses that the river is highly polluted for long stretch of the watercourse, thus implying that selected

system of treatment method must be put in place.

### 4. CONCLUSION

From the result obtained therefore, it was revealed that effluent produced through the slaughterhouse is highly polluted and might have an unfavorable stimulus on the stream water were it is being disposed into without a sort of treatment. The first-order de-oxygenation constant for the river which was used to simulate the river BOD<sub>5</sub> conditions downstream was 0.0014day<sup>-1</sup> at a flow velocity of 0.32 m/s.

In this study, finite element modeling approach was utilized for the extrapolation of BOD<sub>5</sub> in Apa Mmini stream. The outcomes of the simulation showed that the Apa Mmini stream is highly polluted with BOD<sub>5</sub> of 111.8mg/l which is far above the FME<sub>Env</sub> recommended standard of 50 mg/l.

### COMPETING INTERESTS

Authors have declared that no competing interests exist.

### REFERENCES

1. Zhai X, Xia J, Zhang Y. Water quality variation in the highly disturbed Huai River Basin, China from 1994 to 2005 by multi-statistical analyses. *Sci The Total Environ.* 2014;496:594-606.
2. Nafarnda WD, Yaji A, Kubkomawa HI. Impact of slaughterhouse waste on aquatic life: A case study of Yola slaughterhouse. *Global Journal of Pure Applied Sci.* 2006;12:31–33.
3. Adelegan JA. Environmental policy and slaughterhouse waste in Nigeria. *Proceedings of the 28<sup>th</sup> WEDC Conference.* Calcutta, India. 2002; 3-6.
4. Diesch SL. Disease transmission of water-borne organisms of animal origin, Agricultural practices and water quality, Iowa State University Press, Ames, Iowa. 1975;265-285.
5. Hollon BF. Water quality in a stream receiving dairy feedlot effluent. *Journal of Environ. Quality.* 1982;11(1):5-9.
6. Nwaogazie IL. Finite element modelling of engineering systems. 2<sup>nd</sup> edition, Special publishers, Enugu. 2008;115-135.
7. Segerlind JL. Applied finite element analysis. John Wiley & Sons. Inc, New York; 1984.
8. Clough RW. Early history of the finite element method from the view point of a pioneer. *International Journal for Numerical Methods in Engineering.* 2004; 60(1):283-287.
9. Zienkiewicz OC. The birth of the finite element method and of computational mechanics. *International Journal for Numerical Methods in Engineering.* 2004; 60(1):3-10.
10. Dahlquist G, Bjorck A. Numerical methods, Prentice-Hall, Englewood Cliffs, NJ; 1974.

© 2020 Daniel et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

*Peer-review history:*  
*The peer review history for this paper can be accessed here:*  
<http://www.sdiarticle4.com/review-history/54653>