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A Novel Approach for Sphere Decoder MIMO System

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Research Article

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Abstract

In sphere decoding techniques, it was seen that the generalized sphere decoder algorithms have been applied to decode the MIMO systems. The transmitted vector is determined by decoding a sequence of determined sub problems.In this paper, the proposed sphere decoder has promised considerable performance as compared to the generalized sphere decoder. The proposed sphere decoding algorithm follows an adaptive radius selection approach for reducing the computational complexity as compared to the generalized sphere decoding algorithm and with the ideal ML decoder which applies on the QPSK signaling. Also it has proven that the proposed sphere decoder has given the performance near optimal like the ML decoder. The performance comparison is done under the Flat Rayleigh fading environment. We substantiate our new proposed reduced complexity near optimal sphere decoding algorithm with simulations.

Keywords: Multiple Input Multiple Output (MIMO), Intersymbol Interference (ISI), Zero Forcing (ZF), Minimum Mean-Squared Error (MMSE), Fixed –Complexity Sphere Decoder (FSD).

1 Introduction

The pursuit of high speed wireless data services has made the communication researchers relatively active. The limitations of wireless medium are a challenge to the researchers as demand is continuously increasing for whatever bandwidth available. This has led to the search for a reliable and high data rate communication system. In multi-antenna system, space-time [1,2] (along with traditional error-correcting) codes are often employed at the transmitter to induce diversity. Moreover to secure the highly reliable data transmission, special attention should be given to the design of receiver antenna. The received signal is the combination of the transmitted signals perturbed by noise, Inter Symbol Interference & Inter User Interference.

MIMO system have attracted much attention for more than a decade because they provide high data rate transmission over wireless channels and theoretically show considerably improved spectral efficiencies. An optimal performance can be obtained by implementing the ML decoder

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but its exponential complexity makes it unrealizable in practical systems when a large number of antennas and higher order constellations are used.

In order to attain ML performance at reduced complexity, a multichannel equalizer is used to suppress ISI and IUI. By using Linear or nonlinear different equalizers like ZF, MMSE and V-BLAST, we realize better performance (in terms of high SNRs) in nonlinear equalizers than linear equalizers. In this paper, we select initial radius value in Sphere Decoder which gives less complexity than the ML decoder. This scheme is based on integer lattice theory. The concept behind the sphere decoding [3] is to limit the count of possible code words by considering only those which are within a sphere centered at the received signal vector. Or in a simple way to search the closest lattice point to the received signal within a sphere radius is sphere decoding where each code word is represented by a lattice point in a lattice field [4].

By tree search approach for lattice points inside a hyper-sphere from Fig. 1, we reduce complexity. It has two kinds of approach: Fincke-Pohst strategy [5,6] and Schnorr-Euchner strategy [7,8]. The Fincke-Pohst(FP) strategy introduces sphere decoding algorithm, where one major issue is selection of initial radius. The viterbo-boutros (VB) [9] modified initial radius with adaptive updating of the sphere radius as a new solution is found. The Schnorr-Euchner (SE) strategy is an alteration of the FP and VB algorithm, in which nearest nodes of each layer are extent in a zigzag ordering. In this method initial radius is selected with the closest middle point, while in FP method, the allowable nodes are searched without any ordering.

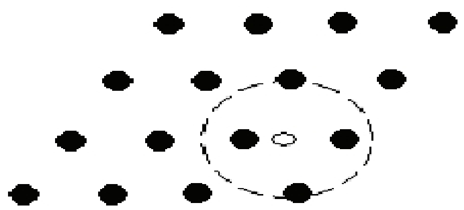

Fig. 1. Idea behind sphere decoder

Starting with Maximum Likelihood receiver which is an optimal receiver, it solves for estimating symbol vector as

$$\tilde{x} = \arg\min\|y - Hx\|^2 \tag{1}$$

$\tilde{x}$is the estimated symbol vector. The most probable transmitted signal vector can be searched by the ML decoder through all the vector constellations.

2 MIMO System Model

For Lattice Representation of Multiple Antenna Systems, let us consider a multiple antenna system (MIMO) with $t_r$ansmit antennas and $r_e$ceiving antennas ($n_T \le n_R$ [1] and [2]). By using a QPSK modulation technique for Rayleigh fading channel for baseband signal vector transmitted during each symbol period is, $\tilde{x} = [\widetilde{x_1}, \widetilde{x_2}, \dots \dots, \widetilde{x_n}]^T$ in which each component is independently drawn from a complex constellation. Flat fading channel is modeled between each transmit and receive antenna resulting in an n_Rx n_T channel transfer function $\widetilde{H}$ whose elements are $\widetilde{h}_{ij}$ which represent the complex transfer functions from the j^{th} transmit antenna to the i^{th} receive antenna. Also, the elements $\tilde{h}_{ij}$are independent identically distributed (i.i.d) complex Gaussian noise with zero mean. The received vector during each symbol period is can be written as

$$\tilde{y} = \tilde{H}\tilde{x} + \tilde{n} \tag{2}$$

For, lattice representation, transformation of the complex matrix equation into a real matrix equation,

y = Hx + n, where

$$Y = [\mathrm{Re}\,\{\tilde{\boldsymbol{y}}^{\boldsymbol{T}}\} \quad \mathrm{Im}\,\{\tilde{\boldsymbol{y}}^{\boldsymbol{T}}\}\,]^{T}, \tag{3}$$

$$X = [\mathrm{Re}\,\{\tilde{\boldsymbol{x}}^{\boldsymbol{T}}\} \quad \mathrm{Im}\,\{\tilde{\boldsymbol{x}}^{\boldsymbol{T}}\}]^{T}, \tag{4}$$

$$H = \begin{bmatrix} \mathrm{Re}\{\tilde{h}\} & -\mathrm{Im}\{\tilde{h}\} \\ \mathrm{Im}\{\mathrm{h}\} & \mathrm{Re}\{\tilde{h}\} \end{bmatrix} \tag{5}$$

$$N = [\mathrm{Re}\,\{\tilde{\boldsymbol{n}}^{\boldsymbol{T}}\} \quad \mathrm{Im}\,\{\tilde{\boldsymbol{n}}^{\boldsymbol{T}}\}]^{T} \tag{6}$$

The MIMO system is can be modeled on the linear model which is

$$y = Hx + n \tag{7}$$

Where, $x \in R^m$, y, $z \in R^n$ denotes the channel input, output and the noise (i.e. zero mean Gaussian random distribution), and $H \in R^{n \times m}$ is the channel matrix of full column rank.

3 Sphere Decoder

Sphere Decoder is an efficient strategy or computes all the lattice points within a sphere with a certain radius [10]. This enumeration strategy was first introduced in digital communications by Viterbo and Biglieri but finally Hassibi and Vikalo presented the General Sphere Decoder (GSD) as a low complexity to find out the least squares solution and hence as a low complexity detector for MIMO systems. According to Vikalo the expected complexity of least square problem has been averaged over the noise and over the lattice. A closed form expression is been given for the infinite and finite lattice.For a wide range of signal-to-noise ratios (SNRs) and numbers of antennas, the expected complexity is polynomial. Vikalo and Hassibi have also worked on the applications of sphere decoding to detection in multi antenna systems.

In communication system the given vector is not arbitrary but it is an unknown lattice point that has been perturbed by an additive noise vector with known statistical properties. Therefore, the complexity of finding the closest lattice point for any given algorithm should, in fact, be viewed as a random variable. They gave a closed form of expressions which demonstrated over wide range of SNR, and shows that the expected complexity of sphere decoding is a polynomial, implies practical feasibility of sphere decoding in many applications.

To get the transmitted signal i.e. x at the receiver, the receiver has to solve the following minimization problem which is called NP-hard $\min_x \|y - Hx\|^2$. The searching for a vector of a closest lattice point search problem (CLPS) is called as decoding.

For setting the initial radius value the deterministic approach is used. Firstly, the QR decomposition method is used to decompose the channel matrix.

H = QR, where H is a channel matrix, Q is ann_R x n_T unitary matrix ($Q^HQ = I$) and R is an upper triangular matrix of n_Tx n_T .

Initially we have,

$$\min\|Hx - y\|^2 \tag{8}$$

By applying QR decomposition to the equation (12), we get

$$\min\|Rx - \tilde{y}\|^2 \tag{9}$$

The triangular system $Rx = \tilde{y}$ is solved to obtain x, $x \in R^m$ .x can be solved by $x = R^{-1}y$, where R^{-1} is the inverse of the matrix R. As the entries of x are real, by rounding off to their nearest integers we can obtain the lattice point $\tilde{x}$, $\tilde{x} \in Z^m$

$$\tilde{x} = \text{roundoff}(x) \tag{10}$$

Thus the initial radius r can be computed by setting it to the Euclidean distance between $R\tilde{x}$ and the vector $\tilde{y}$

$$r = \|R\tilde{x} - \tilde{y}\| \tag{11}$$

where, $\tilde{x}$ is the Babai estimate.

4 Fixed Complex Sphere Decoder

The main idea behind the sphere decoder is to reduce the computational complexity of the maximum likelihood detection by searching over only those points of the lattice that lies within a hyper sphere of radius r around the received signal. To improve the performance of the sphere decoder a new decoding algorithm [11], known as fixed complexity Sphere decoder has been proposed by searching, the noise level independently, over only a fixed number of lattice points Hx, generated by constellation, C, around the received point ras shown in Fig. 2.

In Fixed Complexity Sphere Decoder, if more points are to be searched, the performance will be closer to that of the original Sphere Decoder but it result in increase of required computational power. That makes the FSD suitable for reconfigurable architectures where the number of candidates can be made.

The fixed-complexity sphere decoder consist of full expansion stage followed by single expansion stage to overcome the drawback of searching only limited number of paths in the search tree.

Fixed complexity SD can achieve a performance near to ML, but its complexity is quiet high in large MIMO system.

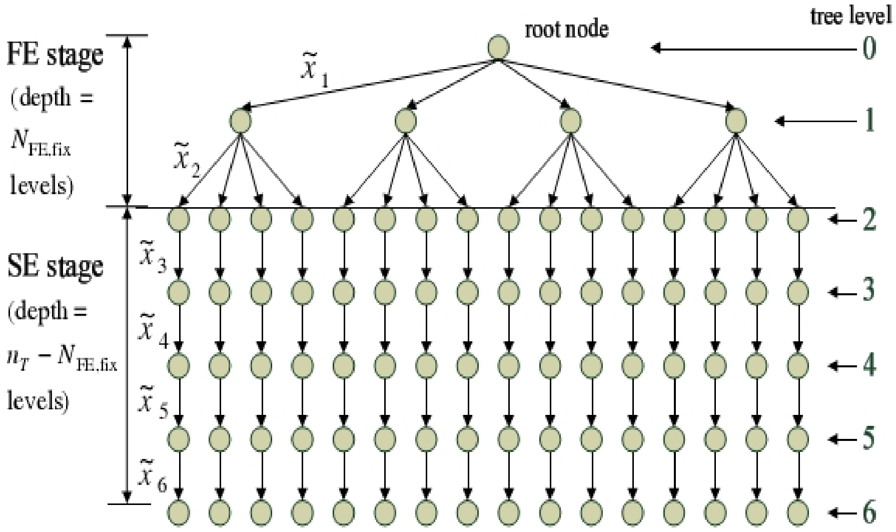

Fig. 2. Search tree of fixed complexity sphere decoder

The nodes are distributed over $N_{FE,fix}$ +1 levels, numbered from the root node n_0 at level 0 to the leaf nodes at level $N_{FE,fix}$. Non-leaf nodes are those at levels 0 through $N_{FE,fix}$ -1.

5 K-Best Sphere Decoder

From Fig. 3 the K-best sphere decoder (KSD) for spatial multiplexing multiple-input multiple-output (MIMO) detection has received significant attention recently because of its throughput and parallel implementation. The K- best sphere decoder is also known as the M-algorithm or as a beam search. Instead of a depth-first tree traversal, the K-best SD performs a breadth-first search and retains only K best nodes at each layer. In K-best SD to maintain a constant throughput, at each level of the tree, K best nodes are selected to be expanded to the next level. Any other nodes are discarded. This process essentially involves two tasks. First is to find the center at that specific tree level, and second is to find the partial branch metric or cost of extension to a node. The extension and selection process consists of several operations including path extension which is the metric computation for newly extended paths, path comparison involving the comparison with previously extended path and path purge which is the removal of a path exceeding any pre-defined bound. The speed and power bottle neck of the K-best algorithm arises mainly from the parallel execution of all said operations at each level.

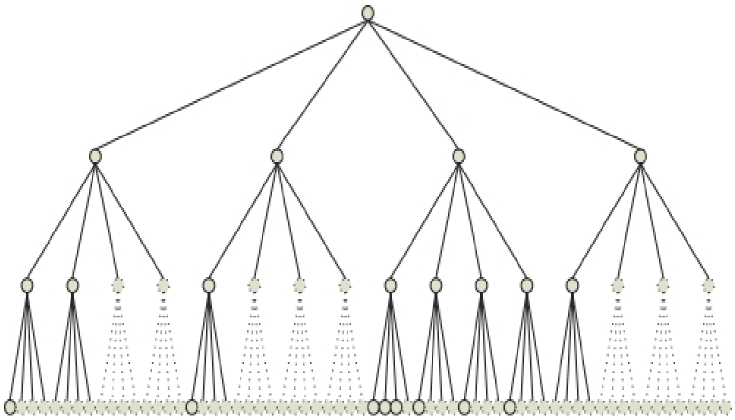

Fig. 3. K-best sphere decoder algorithm

K-best SD visits all child nodes of the kept nodes and keeps a fixed number of nodes for each layer in the search tree no matter how many nodes are visited. At the end of each layer search, all visited nodes should be ranked in increasing order according to their partial Euclidean distance. In the above figure the solid line points to the visited nodes which indicate the kept node and the dotted node. As a result, the number of visited nodes increases linearly with the layer number.

6 Proposed Sphere Decoder

The K-best Algorithm [12] employs breadth-first search instead of depth-first Search. For Sphere decoding the sphere radius is fixed for one execution because we reach only leaf nodes in the end. So we have to find an appropriate radius search's-best Algorithmretains only the best nodes of each layer anyway, the search radius may not be important to obtain the (approximated) ML solution, but it helps reducing the number of calculated nodes in each layer.

The proposed SD consists of a search through a small subset of the complete transmit constellation. When the initial sphere radius *d* is sufficiently large, the algorithm achieves its maximal complexity. When it is smaller, the complexity is reduced with the degradation in performance due to the lost lattice points outside the radius.

For the tree search process, the conventional KSD sorts all the child nodes based on their partial costs, and selects the *K* best paths. In the proposed SD, instead of choosing exactly *K* nodes, we keep the additional nodes whose costs are close to the cost of the K^{th} node, f_K. The main challenge for the proposed SD is the, if α is large more nodes will be visited and the complexity increases while if α is too small the performance improvement is limited.

7 Algorithm

The proposed Sphere Decoder Algorithm is as follows-

Input: an nxn lower-triangular matrix H with positive Diagonal elements and an n-dimensional vector
X€ R^n to decode in the latticeπ (H^{-1})
Output: an n-dimensional vector u'€ Z^n such that u'H-1 is a lattice point that is closest to x.

Input: α, K, x, H, d

```
n := the size of H;  bestdisk := ∞(Channel Matrix of MxN Channel)
k := n ; disk_k =0                    (dimension of examined layer)
e_k := xH ; u_k := [e_kk]               (examined lattice point)
y := (e_kk-u_k)/h_kk
step_k := sgn*(y)   (calculate and shift to next layer)
<loop>
newdist :=dist_k +y^2
ifnewdist< bestdist then {
if k≠1 then {
        e_k-1, i :=e_ki-yh_kifor i=1,…, k-1
        k := k-1   (shift to next layer)
        disk_k := newdist
        u_k := [e_kk] ;        y := (e_kk-u_k)/h_kk
        step_k := sgn*(y)
} else {
        U' := u ;
bestdisk:= newdist             (sort all the components in acending order)
        k := k+1
        u_k := u_k + step_k
        y := (e_kk-u_k)/h_kk
        step_k := -step_k - sgn*( step_k)
}
} else {
        Ifk=n then return u' (and exit)
        else {
        k :=k+1 ;          u_k := u_k + step_k
        y := (e_kk-u_k)/h_kk
        step_k := -step_k - sgn*( step_k)
}
}
goto <loop>
```

In the proposed SD, instead of choosing exactly K nodes, we keep the additional nodes whose costs are close to the cost of the K^{th} node, f_K. For example, at the i^{th} level (where i =1, 2,….. m, m =2N), suppose that the nodes are sorted, if the cost difference between the K^{th} node and the $(K+r)^{th}$ node (r =1, 2...) is less than α, all K+r nodes are retained. The choice of α is the main challenge of the proposed SD, If α is too large, then more nodes are visited and the complexity increases; while if α is too small, the performance is limited compared to the conventional KSD. Depending on the parameterization of α, a flexible performance-complexity trade-off could be achieved. In the proposed algorithm, for a 4x4 MIMO system with noise variance σ^2, we set α in such a way that we achieve its value 0.25 σ^2.

8 Simulation Results

In this part, we have discussed simulation results of the proposed scheme in terms of the bit error rate by using MAT-LAB program. In simulation results we use a QPSK modulation technique for wideband receiver based on the four paths Rayleigh fading environment, assuming perfect Channel estimation. The Fig. 4 compares the error performance of the proposed sphere decoder with a generalized sphere decoder for coded QPSK transmission for 4X4 MIMO System.

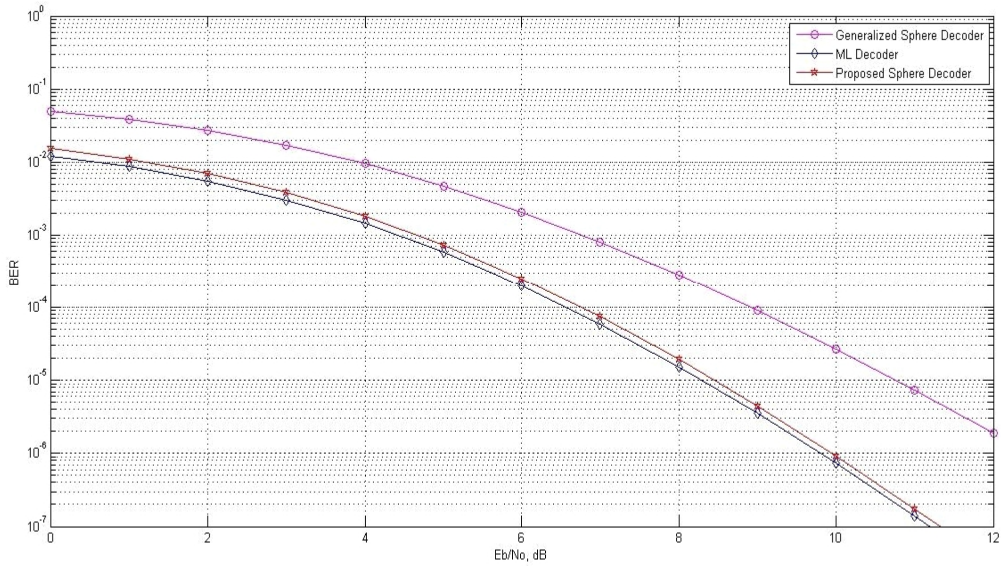

Fig. 4. Bit error performance of generalized sphere decoder, ML decoder and the proposed sphere decoder

As expected, the proposed sphere decoder shows very small BER degradation compared to the generalized sphere decoder. The radius of the hyper sphere is determined by locating the constellation points in the lattice structure and computation complexity is reduced with the help of the differentiation computation method.

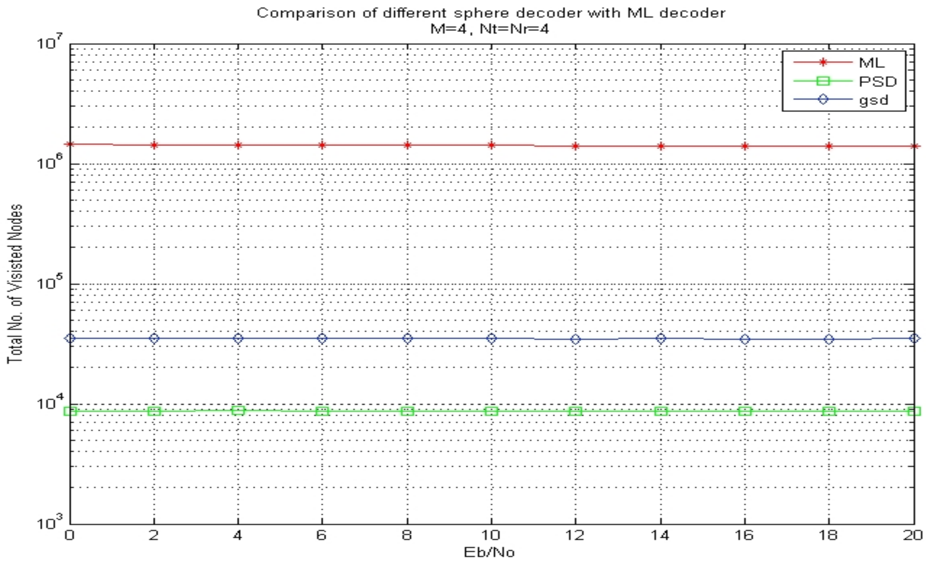

Fig. 5. Comparison of generalized sphere decoder, ML decoder and the proposed sphere decoder

From the Fig. 5, we compare SNR and Total No. of visited nodes (complexity) in Proposed Sphere Decoder and Sphere decoder with ML Decoder. We notice that the complexity of theProposed Sphere Decoder is lower than the Sphere decoder.

Table 1. Summary of the complexity of payload processing for MIMO detection techniques for $N_t = N_r = 4$

	Mean of real multiplication	Mean of real addition	Mean of estimated addition	No. of flops	Max. no. of node visited
ML	255	128	1535	893	$10^{6.5}$
SD	196	155	1746	743	$10^{4.2}$
PSD	148	104	1188	548	$10^{3.9}$

Proposed Sphere Decoder achieves significant reduction in the average complexity as it can be seen in the Table 1 for four transmit and receive antennas. When we consider the mean of real multiplication, there is reduction in the proposed sphere decoder, also when the mean of real addition is calculated, the proposed sphere decoder shows better performance. Also, the numbers of counted flops are decreased in the proposed Sphere Decoder.

9 Conclusion

We propose a sphere decoder algorithm based on the Schnorr – Euchner method, which is compatible pre-processing stage for a sphere decoder. Simulation results show that the proposed sphere decoder performs better than the generalized sphere decoder. Also it is resulting in a significant reduction in the computational cost of sphere decoding stages; especially efficient in the low SNR system and from the graph BER is reduced from 10^{-1} to 10^{-2}. The performance of the proposed sphere decoder is near the ML decoder and it is better than the generalized sphere decoder.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Damen O, Chkeif A, Belfiore JC. Lattice code decoder for space-time codes, IEEE Communication. Letters, May 2000;4:161–163.
Available: http://www.ieeexplore.ieee.org/Communications /10.1109/4234.846498.

[2] Strang G. Linear Algebra and its Application, 2nd ed., New York: Academic Press; 1980.

[3] David Forney JR. G. A bounded-distance decoding algorithm for the leech lattice, with generalizations, IEEE Transactions on Information Theory. August 1989;35(4):906–909.
Available: http://www.ieeexplore.ieee.org/Information Theory, IEEE/10.1109/18.32173.

[4] Azzam L, Ayanoglu E. Reduced Complexity Sphere Decoding via a Reordered Lattice Representation, IEEE Transaction on Communications, Sept. 2009;57(9):2564-2569.
Available:http://www.ieeexplore.ieee.org/Communications/10.1109/TCOMM.2009.09.070238.

[5] Damen MO, Abed-Meraim K, Belfiore JC. Generalized sphere decoder for asymmetrical space-time communication architecture, Electronics Letters. Jan. 2000;36(2):166-167.
Available: http://www.ieeexplore.ieee.org/Electronics/ 10.1049/el: 20000168.

[6] Golden GD, Foschini GJ, Valenzuela RA, Wolniansky PW. Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture, Electronics letters. Jan. 1999;35(1):14-15.
Available: http://www.ieeexplore.ieee.org/Electronics Letters / 10.1049/el: 19990058.

[7] Telatar E. Capacity of multi-antenna Gaussian channels, European Transactions on Telecommunications.1999;6(10):585-595.
Available: http://onlinelibrary.wiley.com/doi/10.1002/ett.4460100604.

[8] Gerard J. Foschini. Layered space-time architecture for wireless communication in a fading environment when using multiple antennas, Bell Laboratories Technical Journal. August 1996; 1(2):41–59.
Available: http://www.ece.mtu.edu/faculty/ztian/ee5535s08/4-1-BLAST.

[9] Fincke U, Pohst M. Improved methods for calculating vectors of short length in a Lattice, including a complexity analysis Mathematics of Computation. April 1985; 44(170):463–471.
Available: http://dx.doi.org/10.1090/S0025-5718-1985-0777278-8.

[10] Conway J, Sloane N. Sphere Packing, Lattices and Groups, 3rd ed Springer Verlag; 1998.

[11] Jaldén J, Elia P. Sphere Decoding Complexity Exponent for Decoding Full-Rate Codes Over the Quasi-Static MIMO Channel, IEEE Transaction on Information Theory, Sept. 2012;(58)9:5485–5803.
Available: http://www.ieeexplore.ieee.org/Information Theory/10.1109/TIT.2012.2203581.

[12] Ghasemmehdi A, Agrell E. Faster recursions in sphere decoding, IEEE Transaction on Information Theory. Jun. 2011;57(6):3530- 3536.
Available: http://www.ieeexplore.ieee.org/Information Theory/10.1109/TIT.2011.2143830.

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