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Ridge Estimator in Logistic Regression under Stochastic Linear Restrictions

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Authors' contributions

This work was carried out in collaboration between both authors. Both the authors designed the study. Author NV managed literature searches, performed the statistical analysis, and wrote the first draft of the manuscript. Author PW supervised the work, and edited the manuscript. Both authors read and approved the final manuscript.

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Abstract

In the logistic regression, it is known that multicollinearity affects the variance of Maximum Likelihood Estimator (MLE). To overcome this issue, several researchers proposed alternative estimators when exact linear restrictions are available in addition to sample model. In this paper, we propose a new estimator called Stochastic Restricted Ridge Maximum Likelihood Estimator (SRRMLE) for the logistic regression model when the linear restrictions are stochastic. Moreover, the conditions for superiority of SRRMLE over some existing estimators are derived with respect to Mean Square Error (MSE) criterion. Finally, a Monte Carlo simulation is conducted for comparing the performances of the MLE, Ridge Type Logistic Estimator (LRE) and Stochastic Restricted Maximum Likelihood Estimator (SRMLE) for the logistic regression model by using Scalar Mean Squared Error (SMSE).

Keywords: Logistic regression; multicollinearity; stochastic restricted ridge maximum likelihood Estimator; mean square error; scalar mean squared error.

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1 Introduction

There are many fields of research where the response variable is binary. For instance, diagnosis of breast cancer (present, absent), vote in election (Democrat, Republican), or mode of travel to work (by car/ by bus) and so-on. The logistic regression plays an important role in predicting the binary outcomes as stated above.

The general form of logistic regression model is given as follow:

$$y_i = \pi_i + \varepsilon_i, \quad i = 1, ..., n \tag{1.1}$$

which follows Bernoulli distribution with parameter π_i as

$$\pi_i = \frac{\exp(x_i'\beta)}{1 + \exp(x_i'\beta)}, \tag{1.2}$$

where x_i is the i^{th} row of X, which is an $n \times (p + 1)$ data matrix with p explanatory variables and β is a $(p + 1) \times 1$ vector of coefficients, ε_i are independent with mean zero and variance $\pi_i(1 - \pi_i)$ of the response y_i. The Maximum likelihood method is the most common estimation technique to estimate the parameter β, and the maximum likelihood estimator (MLE) of β can be obtained as follows:

$$\hat{\beta}_{MLE} = C^{-1}X'\hat{W}Z, \tag{1.3}$$

where $C = X'\hat{W}X$; Z is the column vector with i^{th} element equals $logit(\hat{\pi}_i) + \frac{y_i - \hat{\pi}_i}{\hat{\pi}_i(1-\hat{\pi}_i)}$ and $\hat{W} = diag[\hat{\pi}_i(1 - \hat{\pi}_i)]$, which is an unbiased estimate of β. The covariance matrix of $\hat{\beta}_{MLE}$ is

$$Cov(\hat{\beta}_{MLE}) = \{X'\hat{W}X\}^{-1}. \tag{1.4}$$

There are situations where the explanatory variables have strong inter-relationship called multicollinearity. This causes inaccurate estimation of model parameters. As a result, the estimates have large variances and large confidence intervals, which produces inefficient estimates. One way to deal with this problem is called the ridge regression, which was first introduced by [1]. To overcome the problem of multi-collinearity in the logistic regression, many authors suggested different estimators alternative to the MLE. First, [2] proposed the Ridge Logistic Regression estimator for logistic regression model. Later, Principal Component Logistic Estimator (PCLE) by [3], the Modified Logistic Ridge Regression Estimator (MLRE) by [4], Liu Estimator by [5], Liu-type estimator by [6] and Almost unbiased ridge logistic estimator (AURLE) by [7] have been proposed. Recently, [8], proposed some new methods to solve the multicollinearity in logistic regression by introducing the shrinkage parameter in Liu-type estimators.

An alternative way to solve the multi-collinearity problem is to consider parameter estimation with priori available linear restrictions on the unknown parameters, which may be exact or stochastic. That is, in some practical situations there exists different sets of prior information from different sources like past experience or long association of the experimenter with the experiment and similar kind of experiments conducted in the past. If the exact linear restrictions are available in addition to logistic regression model, many authors proposed different estimators for the respective parameter β. [9] introduced a restricted maximum likelihood estimator (RMLE) by incorporating the exact linear restriction on the unknown parameters. [10] proposed a new estimator called Restricted Liu Estimator (RLE) by replacing MLE with RMLE in the logistic Liu estimator. However, RLE estimator did not satisfy the linear restriction. Consequently, [11] proposed a Modified Restricted Liu Estimator in logistic regression, which satisfies the linear restrictions. Later, [12] investigated the theoretical results about the mean squared error properties of the restricted estimator compared to MLE, RMLE and Liu estimator. When the restriction on the parameters are stochastic, [13] recently proposed a new estimator called Stochastic Restricted Maximum Likelihood Estimator (SRMLE) and derived the superiority conditions of SRMLE over the estimators Logistic Ridge Estimator (LRE), Logistic Liu Estimator (LLE) and RMLE.

In this paper, we propose a new estimator called Stochastic Restricted Ridge Maximum likelihood Estimator (SRRMLE) when the linear stochastic restrictions are available in addition to the logistic regression model. The rest of the paper is organized as follows, the proposed estimator and its asymptotic properties are discussed in Section 2. In Section 3, the mean square error matrix and the scalar mean square error for this new estimator are obtained. Section 4 describes the theoretical performance of the proposed estimator over some existing estimators. The performance of the proposed estimator with respect to Scalar Mean Squared Error (SMSE) is investigated by performing a Monte Carlo simulation study in Section 5 . The conclusions of the study is presented in Section 6.

2 A Proposed New Estimator

In the presence of multicollinearity in logistic regression model (1.1), [2] proposed the Logistic Ridge Estimator (LRE), which is defined as

$$\begin{aligned} \hat{\beta}_{LRE} &= (X'\hat{W}X + kI)^{-1}X'\hat{W}X\hat{\beta}_{MLE} \\ &= (C + kI)^{-1}C\hat{\beta}_{MLE} \\ &= Z_k\hat{\beta}_{MLE} \end{aligned} \tag{2.1}$$

where $Z_k = (C + kI)^{-1}C$ and k is a constant, $k \geq 0$.

The asymptotic properties of LRE:

$$E[\hat{\beta}_{LRE}] = E[Z_k\hat{\beta}_{MLE}] = Z_k\beta \tag{2.2}$$

$$\begin{aligned} Var[\hat{\beta}_{LRE}] &= Var[Z_k\hat{\beta}_{MLE}] \\ &= Z_kC^{-1}Z_k' \\ &= (C + kI)^{-1}C(C + kI)^{-1} \\ &= Z_k(C + kI)^{-1} \end{aligned} \tag{2.3}$$

Suppose that the following linear prior information is given in addition to the general logistic regression model (1.1).

$$h = H\beta + \upsilon; \quad E(\upsilon) = \mathbf{0}, \quad Cov(\upsilon) = \Psi \tag{2.4}$$

where h is an $(q \times 1)$ stochastic known vector, H is a $(q \times (p + 1))$ of full rank $q \leq (p + 1)$ known elements and υ is an $(q \times 1)$ random vector of disturbances with mean $\mathbf{0}$ and dispersion matrix Ψ, which is assumed to be a known $(q \times q)$ positive definite matrix. Further, it is assumed that υ is stochastically independent of ε, i.e) $E(\varepsilon\upsilon') = 0$.

In the presence of exact linear restrictions on regression coefficients ($\upsilon = 0$ in (2.4)) in addition to the logistic regression model (1.1), [9] proposed the following Restricted Maximum Likelihood Estimator (RMLE).

$$\hat{\beta}_{RMLE} = \hat{\beta}_{MLE} + C^{-1}H'(HC^{-1}H')^{-1}(h - H\hat{\beta}_{MLE}) \tag{2.5}$$

The asymptotic variance and bias of $\hat{\beta}_{RMLE}$,

$$Var(\hat{\beta}_{RMLE}) = C^{-1} - C^{-1}H'(HC^{-1}H')^{-1}HC^{-1} \tag{2.6}$$

$$Bias(\hat{\beta}_{RMLE}) = C^{-1}H'(HC^{-1}H')^{-1}(h - H\hat{\beta}_{MLE}) \tag{2.7}$$

Following [9], [13] proposed an estimator, the Stochastic Restricted Maximum Likelihood Estimator (SRMLE), when the linear stochastic restriction (2.4) is available in addition to the logistic regression model (1.1).

$$\hat{\beta}_{SRMLE} = \hat{\beta}_{MLE} + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(h - H\hat{\beta}_{MLE}) \tag{2.8}$$

The estimator $\hat{\beta}_{SRMLE}$ is asymptotically unbiased.

$$E(\hat{\beta}_{SRMLE}) = \beta \tag{2.9}$$

The asymtotic covariance matrix of SRMLE

$$Var(\hat{\beta}_{SRMLE}) = C^{-1} - C^{-1}H'(\Psi + HC^{-1}H')^{-1}HC^{-1} \tag{2.10}$$

Moreover, it was shown in their paper that the estimator SRMLE is always superior to MLE. However, the estimator SRMLE is superior over RLE, LLE and RMLE under certain conditions. For further development, in this paper, following [13], we introduce a new biased estimator which is called Stochastic Restricted Ridge Maximum likelihood Estimator (SRRMLE), and defined as

$$\hat{\beta}_{SRRMLE} = \hat{\beta}_{LRE} + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(h - H\hat{\beta}_{LRE}) \tag{2.11}$$

Asymptotic Properties of SRRMLE

$$\begin{aligned} E(\hat{\beta}_{SRRMLE}) &= E[\hat{\beta}_{LRE}] + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(H\beta - HE(\hat{\beta}_{LRE})) \\ &= Z_k\beta + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(H\beta - HZ_k\beta) \\ &= [Z_k + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(H - HZ_k)]\beta \end{aligned} \tag{2.12}$$

$$\begin{aligned} Var(\hat{\beta}_{SRRMLE}) &= Var[\hat{\beta}_{LRE} + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(h - H\hat{\beta}_{LRE})] \\ &= Z_k(C + kI)^{-1} + C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C + kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad -2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C + kI)^{-1} \end{aligned} \tag{2.13}$$

3 Mean Square Error Matrix Criteria

To compare different estimators with respect to the same parameter vector β in the regression model, one can use the well known Mean Square Error (MSE) Matrix and/or Scalar Mean Square Error (SMSE) criteria.

$$\begin{aligned} MSE(\hat{\beta}, \beta) &= E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)'] \\ &= D(\hat{\beta}) + B(\hat{\beta})B'(\hat{\beta}) \end{aligned} \tag{3.1}$$

where $D(\hat{\beta})$ is the dispersion matrix, and $B(\hat{\beta}) = E(\hat{\beta}) - \beta$ denotes the bias vector.

The Scalar Mean Square Error (SMSE) of the estimator $\hat{\beta}$ can be defined as

$$SMSE(\hat{\beta}, \beta) = trace[MSE(\hat{\beta}, \beta)] \tag{3.2}$$

For two given estimators $\hat{\beta}_1$ and $\hat{\beta}_2$, the estimator $\hat{\beta}_2$ is said to be superior to $\hat{\beta}_1$ under the MSE criterion if and only if

$$M(\hat{\beta}_1, \hat{\beta}_2) = MSE(\hat{\beta}_1, \beta) - MSE(\hat{\beta}_2, \beta) \geq 0. \tag{3.3}$$

For the proposed estimator SRRMLE:

$$\begin{aligned} Bias(\hat{\beta}_{SRRMLE}) &= E(\hat{\beta}_{SRRMLE}) - \beta \\ &= [Z_k + C^{-1}H'(\Psi + HC^{-1}H')^{-1}(H - HZ_k) - I]\beta \\ &= \delta_4 \quad (say) \end{aligned} \tag{3.4}$$

The Mean Square Error

$$\begin{aligned} MSE(\hat{\beta}_{SRRMLE}) &= D(\hat{\beta}_{SRRMLE}) + B(\hat{\beta}_{SRRMLE})B'(\hat{\beta}_{SRRMLE}) \\ &= Z_k(C + kI)^{-1} + C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C + kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad -2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C + kI)^{-1} + \delta_4\delta_4' \end{aligned} \tag{3.5}$$

The Scalar Mean Square Error

$$SMSE(\hat{\beta}_{SRRMLE}) = trace\{MSE(\hat{\beta}_{SRRMLE})\} \tag{3.6}$$

4 The Performance of the Proposed Estimator

In this section, we describe the theoretical performance of the proposed estimator SRRMLE over some existing estimators: MLE, LRE, and SRMLE with respect to the mean square error sense.

- **SRRMLE Versus MLE**

$$\begin{aligned} MSE(\hat{\beta}_{MLE}) - MSE(\hat{\beta}_{SRRMLE}) &= \{D(\hat{\beta}_{MLE}) - D(\hat{\beta}_{SRRMLE})\} \\ &\quad +\{B(\hat{\beta}_{MLE})B'(\hat{\beta}_{MLE}) - B(\hat{\beta}_{SRRMLE})B'(\hat{\beta}_{SRRMLE})\} \\ &= \{C^{-1} - Z_k(C + kI)^{-1} - C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C + kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad +2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C + kI)^{-1}\} - \delta_4\delta_4' \\ &= \{C^{-1} + 2Z_HHZ_k(C + kI)^{-1}\} \\ &\quad -\{Z_k(C + kI)^{-1} + Z_H[\Psi + HZ_k(C + kI)^{-1}H']Z_H' + \delta_4\delta_4'\} \\ &= M_1 - N_1 \end{aligned} \tag{4.1}$$

where $Z_H = C^{-1}H'(\Psi + HC^{-1}H')^{-1}$, $M_1 = C^{-1} + 2Z_HHZ_k(C + kI)^{-1}$ and $N_1 = \{Z_k(C + kI)^{-1} + Z_H[\Psi + HZ_k(C + kI)^{-1}H']Z_H' + \delta_4\delta_4'\}$. One can obviously say that $Z_k(C + kI)^{-1} + Z_H[\Psi + HZ_k(C + kI)^{-1}H']Z_H'$ and M_1 are positive definite and $\delta_4\delta_4'$ is non-negative definite matrices. Further by Theorem 1 (see Appendix A), it is clear that N_1 is positive definite matrix. By lemma 1 (see Appendix A), if $\lambda_{\max}(N_1 M_1^{-1}) < 1$, then $M_1 - N_1$ is a positive definite matrix, where $\lambda_{\max}(N_1 M_1^{-1})$ is the largest eigen value of $N_1 M_1^{-1}$. Based on the above arguments, the following theorem can be stated.

Theorem 1: The estimator SRRMLE is superior to MLE if and only if $\lambda_{\max}(N_1 M_1^{-1}) < 1$.

- **SRRMLE Versus LRE**

$$\begin{aligned} MSE(\hat{\beta}_{LRE}) - MSE(\hat{\beta}_{SRRMLE}) &= \{D(\hat{\beta}_{LRE}) - D(\hat{\beta}_{SRRMLE})\} \\ &\quad +\{B(\hat{\beta}_{LRE})B'(\hat{\beta}_{LRE}) - B(\hat{\beta}_{SRRMLE})B'(\hat{\beta}_{SRRMLE})\} \\ &= \{(C+kI)^{-1}C(C+kI)^{-1} - \\ &\quad Z_k(C+kI)^{-1} + C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C+kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad -2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C+kI)^{-1}\} \\ &\quad +\{\delta_1\delta_1' - \delta_4\delta_4'\} \end{aligned} \tag{4.2}$$

where $Z_H = C^{-1}H'(\Psi + HC^{-1}H')^{-1}$ and $\delta_1 = E(\hat{\beta}_{LRE}) - \beta$; bias vector of $\hat{\beta}_{LRE}$.
Now consider,

$$\begin{aligned} D(\hat{\beta}_{LRE}) - D(\hat{\beta}_{SRRMLE}) &= \{(C+kI)^{-1}C(C+kI)^{-1}\} - \{Z_k(C+kI)^{-1} \\ &\quad +C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C+kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad -2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C+kI)^{-1}\} \\ &= 2Z_H HZ_k(C+kI)^{-1} - Z_H[\Psi + HZ_k(C+kI)^{-1}H']Z_H' \\ &= M_2 - N_2 \\ &= D_1^*(say) \end{aligned} \tag{4.3}$$

where $M_2 = 2Z_H HZ_k(C+kI)^{-1}$ and $N_2 = Z_H[\Psi + HZ_k(C+kI)^{-1}H']Z_H'$. One can obviously say that M_2 and N_2 are positive definite matrices. By lemma 1, if $\lambda_{\max}(N_2 M_2^{-1}) < 1$, then $D_1^* = M_2 - N_2$ is a positive definite matrix, where $\lambda_{\max}(N_2 M_2^{-1})$ is the the largest eigen value of $N_2 M_2^{-1}$. Based on the above arguments and lemma 2, the following theorem can be stated.

Theorem 2: When $\lambda_{\max}(N_2 M_2^{-1}) < 1$, the estimator $SRRMLE$ is superior to LRE if and only if $\delta_4'(D_1^* + \delta_1'\delta_1)^{-1}\delta_4 \leq 1$.

- **SRRMLE Versus SRMLE**

$$\begin{aligned} MSE(\hat{\beta}_{SRMLE}) - MSE(\hat{\beta}_{SRRMLE}) &= \{D(\hat{\beta}_{SRMLE}) - D(\hat{\beta}_{SRRMLE})\} \\ &\quad +\{B(\hat{\beta}_{SRMLE})B'(\hat{\beta}_{SRMLE}) - B(\hat{\beta}_{SRRMLE})B'(\hat{\beta}_{SRRMLE})\} \\ &= \{Z_k(C+kI)^{-1} + C^{-1}H'(\Psi + HC^{-1}H')^{-1} \\ &\quad [\Psi + HZ_k(C+kI)^{-1}H'](\Psi + HC^{-1}H')^{-1}HC^{-1} \\ &\quad -2C^{-1}H'(\Psi + HC^{-1}H')^{-1}HZ_k(C+kI)^{-1} \\ &\quad -C^{-1} - C^{-1}H'(HC^{-1}H')^{-1}HC^{-1}\} - \delta_4\delta_4' \\ &= \{(C + H'\Psi^1 H)^{-1} + 2Z_H HZ_k(C+kI)^{-1}\} \\ &\quad -\{Z_k(C+kI)^{-1} + Z_H[\Psi + HZ_k(C+kI)^{-1}H']Z_H' + \delta_4\delta_4'\} \\ &= M_3 - N_3 \end{aligned} \tag{4.4}$$

where $M_3 = \{(C + H'\Psi^1 H)^{-1} + 2Z_H HZ_k(C + kI)^{-1}\}$ and $N_3 = \{Z_k(C + kI)^{-1} + Z_H[\Psi + HZ_k(C+kI)^{-1}H']Z'_H + \delta_4\delta'_4\}$. One can obviously say that $Z_k(C+kI)^{-1}+Z_H[\Psi+HZ_k(C+kI)^{-1}H']Z'_H$ and M_3 are positive definite and $\delta_4\delta'_4$ is non-negative definite matrices. Further by Theorem 1, it is clear that N_3 is positive definite matrix. By lemma 1 (see Appendix A), if $\lambda_{\max}(N_3M_3^{-1}) < 1$, then $M_3 - N_3$ is a positive definite matrix, where $\lambda_{\max}(N_3M_3^{-1})$ is the the largest eigen value of $N_3M_3^{-1}$. Based on the above arguments, the following theorem can be stated.

Theorem 3: The estimator $SRRMLE$ is superior to $SRMLE$ if and only if $\lambda_{\max}(N_3M_3^{-1}) < 1$.

Based on the above results one can say that the new estimator SRRMLE is superior to the other estimators with respect to the mean squared error matrix sense under certain conditions. To check the superiority of the estimators numerically, we then consider a simulation study in the next section.

5 A Simulation Study

In this section, we provide the numerical results of the Monte Carlo simulation which is conducted to illustrate the performance of the estimators MLE, LRE, SRMLE and SRRMLE by means of Scalar Mean Square Error (SMSE). Following [14] and [15], we generate the explanatory variables using the following equation.

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{i,p+1}, i = 1, 2, ..., n, \quad j = 1, 2, ..., p \tag{5.1}$$

where z_{ij} are pseudo- random numbers from standardized normal distribution and ρ^2 represents the correlation between any two explanatory variables. Four explanatory variables are generated using (5.1). Four different values of ρ corresponding to 0.70, 0.80, 0.90 and 0.99 are considered. Further for the sample size n, four different values 25, 50, 75, and 100 are considered. The dependent variable y_i in (1.1) is obtained from the Bernoulli(π_i) distribution where $\pi_i = \frac{\exp(x'_i\beta)}{1+\exp(x'_i\beta)}$. The parameter values of $\beta_1, \beta_2, ..., \beta_p$ are chosen so that $\sum_{j=1}^{p} \beta_j^2$ and $\beta_1 = \beta_2 = ... = \beta_p$.

Moreover, we choose the following restrictions.

$$H = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad h = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \text{ and } \Psi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.2}$$

Further for the ridge parameter k, some selected values are chosen so that $0 \le k \le 1$.
The simulation is repeated 2000 times by generating new pseudo- random numbers and the simulated SMSE values of the estimators are obtained using the following equation.

$$S\hat{MSE}(\hat{\beta}^*) = \frac{1}{2000}\sum_{s=1}^{2000}(\hat{\beta}_s - \beta)'(\hat{\beta}_s - \beta) \tag{5.3}$$

where $\hat{\beta}_s$ is any estimator considered in the s^{th} simulation. The results of the simulation are reported in Tables 5.1 - 5.16 (Appendix C) and also displayed in Figures 5.1 - 5.4 (Appendix B). According to Figures 5.1 - 5.4, it can be observed that in general, increase in degree of correlation between two explanatory variables ρ inflates the estimated SMSE of all the estimators and increase in sample size n declines the estimated SMSE of all the estimators. The performance of MLE is poor for all situations considered in the simulation. Especially, increasing the degree of correlation poorly affects the performance of MLE. Further, when $0 \leq k \leq 1$ and $\rho = 0.7,\ 0.8$ the new estimator SRRMLE has smaller SMSE compared to all the other estimators MLE, LRE, and SRMLE with respect to all samples of size n= 25, 50, 75 and 100. Further, it was noted from the simulation results, the estimator SRRMLE dominates the other estimators with respect to the mean square error sense when $\rho = 0.9$ and $\rho = 0.99$ except the following conditions; $k \geq 0.4784$ with $\rho = 0.99$ & $n = 100$, $k \geq 0.4125$ with $\rho = 0.99$ & $n = 75$, $k \geq 0.3361$ with $\rho = 0.99$ & $n = 50$, $k \geq 0.2383$ with $\rho = 0.99$ & $n = 25$ and $k \geq 0.6966$ with $\rho = 0.90$ & $n = 25$. In these circumstances LRE is superior to all other estimators.

6 Concluding Remarks

In this paper, we introduced the Stochastic Restricted Ridge Maximum Likelihood Estimator (SRRMLE) for logistic regression model when the linear stochastic restriction is available. The performances of the estimators SRRMLE over MLE, LRE, and SRMLE were investigated by performing a Monte Carlo simulation study. It is noted that, increasing degree of correlation makes an increase in the SMSE values of all estimators. Results show, when $0 \leq k \leq 1$ and $\rho = 0.7,\ 0.8$, the proposed estimator SRRMLE is superior over the other estimators for all samples of size n= 25, 50, 75 and 100. It was also noted that the estimator LRE has smaller SMSE compared to the other estimators for some k values related to different ρ and n.

Competing Interests

The authors declare that they have no competing interests.

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Appendix A

Theorem 1: Let $A : n \times n$ and $B : n \times n$ such that $A > 0$ and $B \geq 0$. Then $A + B > 0$. ([16])

Lemma 1: Let the two $n \times n$ matrices $M > 0$,$N \geq 0$, then $M > N$ if and only if $\lambda_{\max}(NM^{-1}) < 1$. ([17])

Lemma 2: Let $\tilde{\beta}_j = A_j y$, $j = 1, 2$ be two competing homogeneous linear estimators of β. Suppose that $D = Cov(\tilde{\beta}_1) - Cov(\tilde{\beta}_2) > 0$, where $Cov(\tilde{\beta}_j)$, $j = 1, 2$ denotes the covariance matrix of $\tilde{\beta}_j$. Then $\Delta(\tilde{\beta}_1, \tilde{\beta}_2) = MSEM(\tilde{\beta}_1) - MSEM(\tilde{\beta}_2) \geq 0$ if and only if $d_2'(D + d_1'd_1)^{-1}d_2 \leq 1$, where $MSEM(\tilde{\beta}_j)$, d_j; $j = 1, 2$ denote the Mean Square Error Matrix and bias vector of $\tilde{\beta}_j$, respectively. ([18])

Appendix B

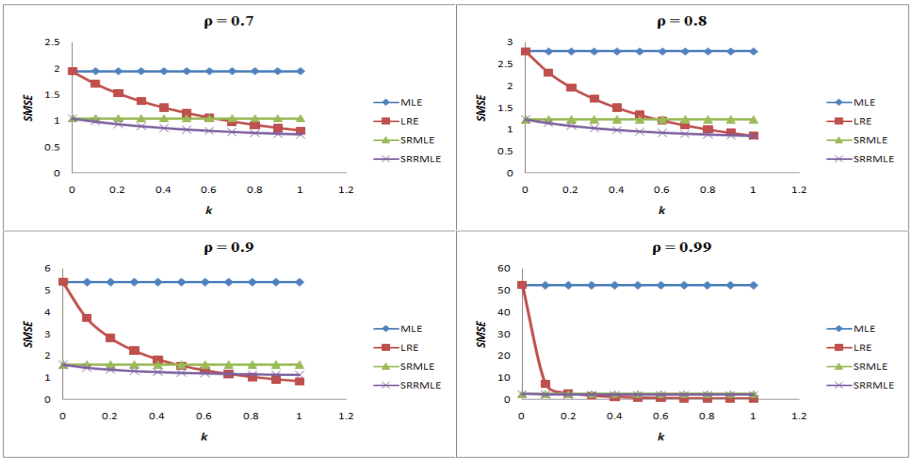

Fig. 5.1. Estimated SMSE values for MLE, LRE, SRMLE and SRRMLE for n = 25

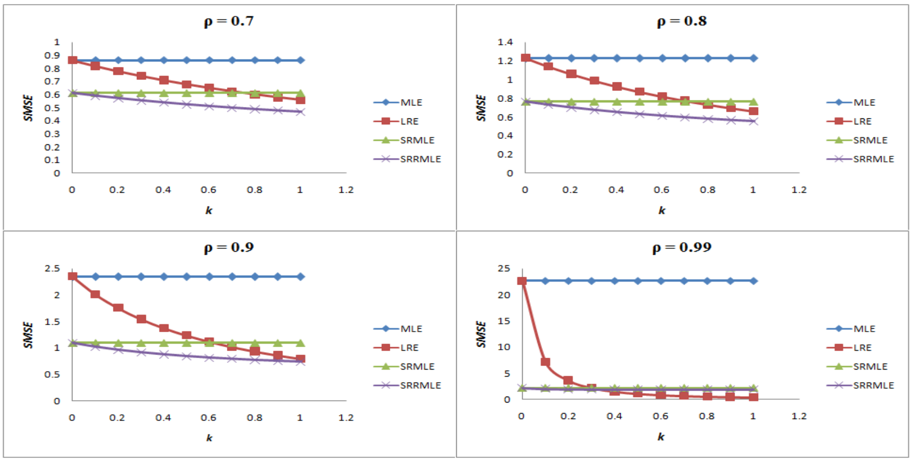

Fig. 5.2. Estimated SMSE values for MLE, LRE, SRMLE and SRRMLE for n = 50

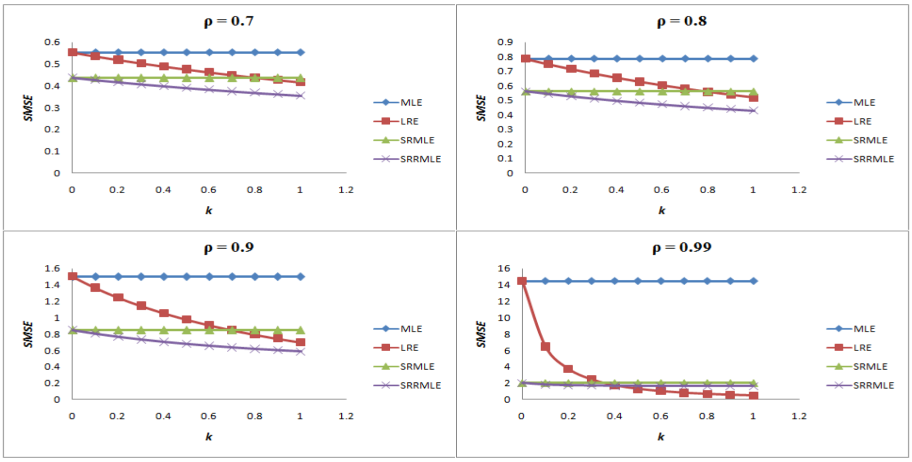

Fig. 5.3. Estimated SMSE values for MLE, LRE, SRMLE and SRRMLE for n = 75

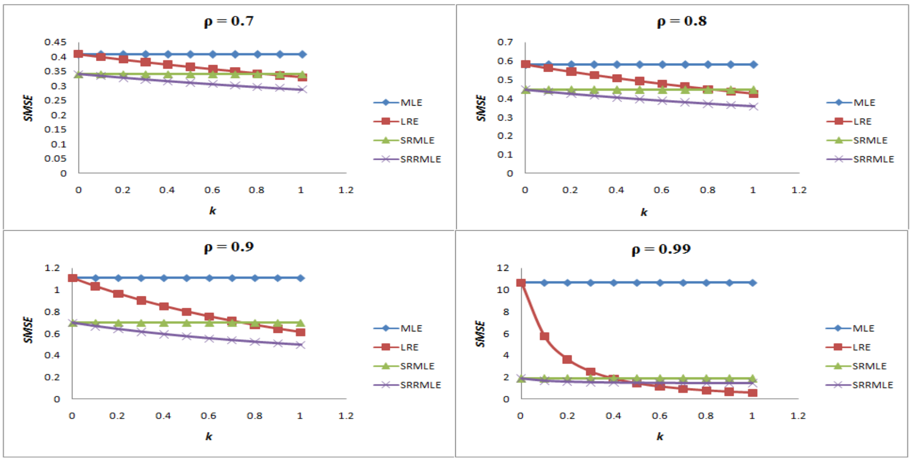

Fig. 5.4. Estimated SMSE values for MLE, LRE, SRMLE and SRRMLE for n = 100

Appendix C

Table 5.1. The estimated MSE values for different k when $n = 25$ and $\rho = 0.70$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464	1.9464
LRE	1.9464	1.7115	1.5261	1.3760	1.2519	1.1480	1.0597	0.9841	0.9187	0.8618	0.8120
SRMLE	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449	1.0449
SRRMLE	1.0449	0.9855	0.9366	0.8958	0.8614	0.8321	0.8069	0.7852	0.7664	0.7500	0.7359

Table 5.2. The estimated MSE values for different k when $n = 25$ and $\rho = 0.80$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913	2.7913
LRE	2.7913	2.3075	1.9612	1.7006	1.4975	1.3351	1.2027	1.0931	1.0010	0.9230	0.8563
SRMLE	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325	1.2325
SRRMLE	1.2325	1.1483	1.0837	1.0325	0.9910	0.9569	0.9284	0.9045	0.8843	0.8670	0.8524

Table 5.3. The estimated MSE values for different k when $n = 25$ and $\rho = 0.90$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804	5.3804
LRE	5.3804	3.7389	2.8255	2.2437	1.8431	1.5527	1.3340	1.1646	1.0304	0.9221	0.8335
SRMLE	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915	1.5915
SRRMLE	1.5915	1.4518	1.3622	1.2996	1.2535	1.2183	1.1909	1.1690	1.1515	1.1373	1.1256

Table 5.4. The estimated MSE values for different k when $n = 25$ and $\rho = 0.99$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691	52.3691
LRE	52.3691	6.9933	2.9006	1.6371	1.0809	0.7863	0.6114	0.4990	0.4227	0.3685	0.3289
SRMLE	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583	2.4583
SRRMLE	2.4583	2.3057	2.2795	2.2682	2.2616	2.2572	2.2541	2.2519	2.2504	2.2494	2.2490

Table 5.5. The estimated MSE values for different k when $n = 50$ and $\rho = 0.70$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628	0.8628
LRE	0.8628	0.8191	0.7794	0.7431	0.7098	0.6793	0.6512	0.6253	0.6014	0.5792	0.5587
SRMLE	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129	0.6129
SRRMLE	0.6129	0.5922	0.5733	0.5560	0.5402	0.5256	0.5123	0.4999	0.4886	0.4781	0.4685

Table 5.6. The estimated MSE values for different k when $n = 50$ and $\rho = 0.80$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291	1.2291
LRE	1.2291	1.1373	1.0569	0.9861	0.9232	0.8672	0.8170	0.7718	0.7310	0.6940	0.6604
SRMLE	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660	0.7660
SRRMLE	0.7660	0.7323	0.7026	0.6762	0.6526	0.6315	0.6126	0.5955	0.5800	0.5660	0.5533

Table 5.7. The estimated MSE values for different k when $n = 50$ and $\rho = 0.90$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496	2.3496
LRE	2.3496	2.0133	1.7515	1.5427	1.3727	1.2321	1.1142	1.0143	0.9288	0.8550	0.7908
SRMLE	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940	1.0940
SRRMLE	1.0940	1.0216	0.9640	0.9170	0.8782	0.8457	0.8181	0.7945	0.7742	0.7565	0.7412

Table 5.8. The estimated MSE values for different k when $n = 50$ and $\rho = 0.99$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486	22.6486
LRE	22.6486	7.1840	3.6292	2.2183	1.5111	1.1053	0.8506	0.6802	0.5605	0.4733	0.4078
SRMLE	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961	2.1961
SRRMLE	2.1961	1.9973	1.9421	1.9181	1.9052	1.8974	1.8924	1.8889	1.8864	1.8846	1.8834

Table 5.9. The estimated MSE values for different k when $n = 75$ and $\rho = 0.70$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536	0.5536
LRE	0.5536	0.5360	0.5195	0.5039	0.4892	0.4753	0.4621	0.4496	0.4378	0.4266	0.4160
SRMLE	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366	0.4366
SRRMLE	0.4366	0.4261	0.4162	0.4069	0.3982	0.3899	0.3821	0.3747	0.3677	0.3611	0.3549

Table 5.10. The estimated MSE values for different k when $n = 75$ and $\rho = 0.80$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871	0.7871
LRE	0.7871	0.7501	0.7160	0.6845	0.6553	0.6282	0.6031	0.5797	0.5579	0.5375	0.5185
SRMLE	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618	0.5618
SRRMLE	0.5618	0.5436	0.5269	0.5114	0.4971	0.4837	0.4714	0.4599	0.4491	0.4391	0.4298

Table 5.11. The estimated MSE values for different k when $n = 75$ and $\rho = 0.90$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018	1.5018
LRE	1.5018	1.3632	1.2446	1.1421	1.0529	0.9747	0.9057	0.8445	0.7899	0.7410	0.6970
SRMLE	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478	0.8478
SRRMLE	0.8478	0.8038	0.7660	0.7330	0.7041	0.6787	0.6562	0.6361	0.6181	0.6020	0.5875

Table 5.12. The estimated MSE values for different k when $n = 75$ and $\rho = 0.99$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586	14.4586
LRE	14.4586	6.4706	3.7324	2.4484	1.7395	1.3058	1.0207	0.8231	0.6805	0.5743	0.4929
SRMLE	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210	2.0210
SRRMLE	2.0210	1.8140	1.7363	1.6980	1.6761	1.6624	1.6532	1.6467	1.6420	1.6385	1.6359

Table 5.13. The estimated MSE values for different k when $n = 100$ and $\rho = 0.70$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091	0.4091
LRE	0.4091	0.3996	0.3905	0.3819	0.3736	0.3656	0.3580	0.3506	0.3436	0.3369	0.3304
SRMLE	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405	0.3405
SRRMLE	0.3405	0.3341	0.3281	0.3222	0.3167	0.3114	0.3063	0.3014	0.2967	0.2923	0.2880

Table 5.14. The estimated MSE values for different k when $n = 100$ and $\rho = 0.80$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813	0.5813
LRE	0.5813	0.5613	0.5424	0.5246	0.5078	0.4919	0.4768	0.4626	0.4491	0.4363	0.4241
SRMLE	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461	0.4461
SRRMLE	0.4461	0.4347	0.4239	0.4137	0.4041	0.3950	0.3864	0.3783	0.3706	0.3633	0.3564

Table 5.15. The estimated MSE values for different k when $n = 100$ and $\rho = 0.90$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084	1.1084
LRE	1.1084	1.0327	0.9651	0.9044	0.8498	0.8003	0.7554	0.7146	0.6772	0.6430	0.6116
SRMLE	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980	0.6980
SRRMLE	0.6980	0.6681	0.6414	0.6173	0.5956	0.5759	0.5579	0.5416	0.5266	0.5129	0.5002

Table 5.16. The estimated MSE values for different k when $n = 100$ and $\rho = 0.99$

	k = 0.0	k = 0.1	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = 1.0
MLE	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602	10.6602
LRE	10.6602	5.7340	3.6198	2.5067	1.8456	1.4200	1.1295	0.9223	0.7692	0.6528	0.5623
SRMLE	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841	1.8841
SRRMLE	1.8841	1.6839	1.5934	1.5442	1.5143	1.4947	1.4812	1.4715	1.4642	1.4587	1.4544

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