Alenezi Transform–A New Transform to Solve Mathematical Problems

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

In this paper, we present a new integral transform called Alenezi-transform in the category of Laplace transform. We investigate the characteristic of Alenezi-transform. We discuss this transform with the other transforms like J, Laplace, Elzaki and Sumudu transforms. We can demonstrate that Alenezi transforms are near to the condition of the Laplace transform. We can explain the new Properties of transforms using Alenezi transform. Alenezi transform can be applied to solve differential, Partial and integral equations.

1 Introduction

Integral transforms techniques are kind of transform to simplify most utilize techniques that transaction with differential equations subject to specific boundary conditions. We can choose a suitable integral transform to convert both differential and integral equations into a solvable algebraic equation. There are some of transforms for solving the differential equations, and these transforms are necessary to solve these equations

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to complete the solutions [1-5]. We can list some of these transformations as shown in tables [1,2] that can demonstrate the functions and those transforms. Many researchers drive some of the integral transforms in the category of Laplace transform like Elzaki, Sumudu, Natural, Pourreza, Aboodh, and J transforms [6-10]. In the Table (1), the definitions for these transforms are registered. These transforms can be utilized for disbanding the different kinds of ordinary, integral, partial and fractional differential equation as in [11-16]. Alenezi et all presented some mathimatical techniques for solving algebraic modules [17-22]. There are Hybrid of the previous transforms with other methods such as the perturbation and Adomian decomposition methods are utilized to find the exact solutions for the different kinds of differential equation [23-29].

Table 1. Definitions of different transforms

Laplace Transform	$L\{\hbar(t)\} = \int_0^\infty \hbar(t)e^{-st}dt$
Elzaki transform	$E\{\hbar(t)\} = s\int_0^\infty \hbar(t)e^{-\frac{t}{s}}dt$
Sumudu transform	$S\{\hbar(t)\} = \frac{1}{s}\int_0^\infty \hbar(t)e^{-\frac{t}{s}}dt$
Natural transform	$n\{\hbar(t)\} = R(s,u) = s\int_0^\infty \hbar(ut)e^{-st}dt$
α-Integral Laplace transform	$L_\alpha\{\hbar(t)\} = \int_0^\infty \hbar(t)e^{-\frac{1}{\alpha}t}dt, \ \alpha \in R_0^+$
Aboodh transform	$A\{\hbar(t)\} = K(s) = \frac{1}{s}\int_0^\infty \hbar(t)e^{-st}dt$
Mohand transform	$m\{\hbar(t)\} = R(s) = s^2\int_0^\infty \hbar(t)e^{-st}dt$
Pourreza transform	$HJ\{\hbar(t)\} = s\int_0^\infty \hbar(t)e^{-st}dt$
Kamal transform	$K\{\hbar(t)\} = G(s) = \int_0^\infty \hbar(t)e^{-\frac{t}{s}}dt$
Sawi transform	$Sa\{\hbar(t)\} = \frac{1}{s^2} = \int_0^\infty \hbar(t)e^{-\frac{t}{s}}dt$
G-transform	$G\{\hbar(t)\} = F(s) = s^\alpha\int_0^\infty \hbar(t)e^{-\frac{t}{s}}dt$

In this paper, we introduced Alenezi integral transform to get the exact solutions of the differential equations. The paper is coordinated as follows. In part 2, we introduce Alenezi integral transform in the category of Laplace transform. In part 3, we match Alenezi integral transform with the other integral transforms in the category of Laplace transform. Alenezi integral transform is utilized to the differential and integral equations to get the exact solutions in part 4. Finally, we summarized the conclusions of my transform in part 4.

2 Alenezi Integral Transform

In this portion, we display Alenezi integral transform that envelope a widely integral transform in the group of Laplace transform.

Definition 1. Let $\hbar(t)$ become an integrable function realized for $t \geq 0, p(s)$ *and* $n(s) \neq 0$ are favorable real functions, we explain Alenezi integral transform $\mathcal{I}(s)$ of $\hbar(t)$ by the formula

$$\mathcal{I}(s) = m(s) \int_0^\infty \hbar(t) e^{-\frac{t}{n(s)}} dt \tag{1}$$

Table 2. Table of alenezi transforms

Function	Alenezi integral transform
1	$\dfrac{m(s)}{n(s)}$
T	$\dfrac{m(s)}{n(s)^2}$
t^α	$\dfrac{\Gamma(\alpha + 1)\, m(s)}{n(s)^{\alpha+1}}$
cos t	$\dfrac{n(s)m(s)}{n(s)^2 + 1}$
sin t	$\dfrac{m(s)}{n(s)^2 + 1}$
sin (at)	$\dfrac{am(s)}{n(s)^2 + a^2}$
e^t	$\dfrac{m(s)}{n(s) - 1}$
$\hbar'(t)$	$n(s)\, \mathcal{I}(s) - m(s)\hbar(0)$

Theorem 1. Let $\hbar(t)$ is differentiable and $m(s)$ and $n(s)$ are positive real functions, then

(I) $\quad T\{\hbar'(t); s\} = n(s)\, \mathcal{I}(s) - m(s)\hbar(0)$ $\hfill (2)$

(II) $\quad T\{\hbar''(t); s\} = n^2(s)\, \mathcal{I}\{\hbar(t); s\} - m(s)n(s)\hbar(0) - m(s)\, \hbar'(0)$ $\hfill (3)$

(III) $\quad T\{\hbar^{(n)}(t); s\} = n^n(s)\, \mathcal{I}\{\hbar(t); s\} - m(s) \sum_{k=0}^{n-1} q^{n-1-k}(s)\hbar^k(0)$ $\hfill (4)$

Proof. (I). In view of (1) we have

$$\mathcal{I}\{\hbar'(t); s\} = m(s) \int_0^\infty \hbar'(t) e^{-n(s)t} dt = m(s)\left[e^{-n(s)t}\hbar(t)\Big|_0^\infty + n(s) \int_0^\infty \hbar(t) e^{-n(s)t} dt \right]$$
$$= n(s)\mathcal{I}\{\hbar(t); s\} - m(s)\hbar(0), \tag{5}$$

To proof (II), we assume $z(t) = \hbar'(t)$ so $\hbar''(t) = z'(t)$ now

$$T\{z'(t); s\} = m(s) \int_0^\infty z'(t) e^{-n(s)t} dt = n(s)\, \mathcal{I}\{z(t); s\} - m(s)z(0) \tag{6}$$

$$= n(s)\mathcal{I}\{\hbar'(t); s\} - m(s)\hbar'(0) = n(s)[n(s)\mathcal{I}\{\hbar(t); s\} - m(s)\hbar(0)] - m(s)\hbar'(0), \tag{7}$$

Theorem 2. (Convolution) Let $\hbar_1(t)$ and $\hbar_2(t)$ have new integral transform $F(s)$. Then the new integral transform of the Convolution of $\hbar_1$ and $\hbar_2$ is

$$\hbar_1 * \hbar_2 = \int_0^\infty \hbar_1(t) * \hbar_2(t - \tau)d\tau = \frac{1}{m(s)}\mathcal{F}_1(s) * \mathcal{F}_2(s). \tag{8}$$

Proof.

$$T\{\hbar_1 * \hbar_2\} = m(s)\int_0^\infty e^{-n(s)t} \int_0^\infty \hbar_1(t) * \hbar_2(t - \tau)dt$$

$$= m(s)\int_0^\infty \hbar_1(\tau)d\tau \int_0^\infty e^{-n(s)t} * \hbar_2(t - \tau)dt$$

$$= m(s)\int_0^\infty e^{-n(s)t} \int_0^\infty \hbar_2(t) * \hbar_2(t - \tau)dt$$

$$= m(s)\int_0^\infty e^{-n(s)t}\hbar_1(\tau)d\tau \int_0^\infty e^{-n(s)t} \hbar_2(t)dt$$

$$= \frac{1}{m(s)}\mathcal{F}_1(s).\mathcal{F}_2(s) \tag{9}$$

3 Solving IVP and Integral Equations by New Transform

In this section, we apply this new integral transform for solving high order IVP with constant coefficient. Also, we applied it to obtain the exact solution of a few types of integral equations and FDE.

4 Solving IVP with Constant Coefficient

Consider the following IVP:

$$X^{(n)}(t) + a_1 X^{(n)}(t) + \cdots + a_n X(t) = g(x) \tag{10}$$

$$X(0) = X_0, \ X'(0) = X_1, \dots, X^{(n-1)}(0) = X_{n-1}. \tag{11}$$

Now we apply a new integral transform, we have:

$$\mathcal{I}\{X^{(n)}(t) + a_1 X^{(n)}(t) + \cdots + a_n X(t)\} = \mathcal{I}\{g(x)\} \tag{12}$$

$$\mathcal{I}\{X^{(n)}(t)\} + a_1 \mathcal{I}\{X^{(n)}(t)\} + \cdots + \mathcal{I}\{a_n X(t)\} = \mathcal{I}\{g(x)\} \tag{13}$$

Example 1. Consider the following third-order ODE

$$X''' + X'' - 6X = 0 \tag{14}$$

$$X(0) = 1, \ X'(0) = 0, \ X''(0) = 5. \tag{15}$$

By applying T on both sides, we have

$$\mathcal{I}\{X''' + X'' - 6X\} = \mathcal{I}\{(0)\} \tag{16}$$

$$\mathcal{I}\{X'''\} + \mathcal{I}\{X''\} - \mathcal{I}\{6X\} = \mathcal{I}\{(0)\} \tag{17}$$

We have;

$$n^3(s) \, \mathcal{I}(s) - m(s)(\, n^2(s) \, X_0 + n(s)X_1 + X_2)+ \, n^2(s) \, \mathcal{I}(s) -$$

$$m(s)(n(s) \, X_0 + X_1 - 6 \, \mathcal{I}(s) = 0 \tag{18}$$

by replacing the initial conditions in above equation, we have

$$[n^3(s) + \, n^2(s) - 6n(s)] \, \mathcal{I}(s) = m(s) \, n^2(s) \, - m(s) + m(s) \, n(s). \tag{19}$$

$$\mathcal{I}(s) = \frac{m(s)(\, n^2(s) - m(s) + m(s)n(s)}{[n^3(s) + \, n^2(s) - 6n(s)]} \underline{\quad} \frac{m(s)}{6n(s)} + \frac{m(s)}{3n(s) + 9} + \frac{m(s)}{2n(s) - 4} \tag{20}$$

by applying $\mathcal{I}^{-1}$ we find the exact solution as:

$$X(t) = \frac{1}{6}\mathcal{I}^{-1}\{\frac{m(s)}{n(s)}\} + \mathcal{I}^{-1}\{\frac{m(s)}{3n(s) + 9}\} + \mathcal{I}^{-1}\{\frac{m(s)}{2n(s) - 4}\} = \frac{1}{6} + \frac{1}{3}e^{-3t} + \frac{1}{2}e^{2t} \tag{21}$$

Example 2. Consider the following third-order ODE

$$X''' + 2X'' + 2X' + 3X = \sin t + \cos t \tag{22}$$

$$X(0) = 1, \ X'(0) = 1, \ X''(0) = 0. \tag{23}$$

By applying $\mathcal{I}$ we have

$$\mathcal{I}\{X''' + 2X'' + 2X' + 3X\} = \mathcal{I}\{\sin t + \cos t\} \tag{24}$$

$$\mathcal{I}\{X'''\} + 2\mathcal{I}\{X''\} - \mathcal{I}\{6X\} = \mathcal{I}\{(0)\} \tag{25}$$

$$n^3(s) \, \mathcal{I}(s) - m(s)(\, n^2(s) \, X_0 + n(s)X_1 + X_2) + 2[n^2(s) \, \mathcal{I}(s) - m(s)(n(s) \, X_0 + X_1] + 2[n(s) \, \mathcal{I}(s) -$$
$$msX0 + 3 \, \mathcal{I}s = ms \ n2s + 1 + nsms \ n2s + 1$$
$$(26)$$

by replacing the initial conditions in above equation, we have

$$[n^3(s) + \, 2n^2(s) + 2n(s) + 3] \, \mathcal{I}(s) = \frac{m(s)}{n^2(s) + 1} + \frac{n(s) \, m(s)}{n^2(s) + 1} + n(s) \, m(s) + 2 \, m(s).$$

by simplification we got.

$$\mathcal{I}(s) = \frac{m(s)}{n^2(s) + 1}. \tag{27}$$

by applying $\mathcal{I}^{-1}$, we find the exact solution as

$$X(t) = \mathcal{I}^{-1}\{\frac{m(s)}{n^2(s) + 1}\} = \sin(t)$$
$$(28)$$

We compare my transform with the laplace transform and found that my transform satisfies the exact solution and give an accurate result like the laplace transform as showen in the next example that demonstrate that my transform satisfies the same results of laplace transform.

Example 3. Solve the Partial differential equation

$$2x \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} = 2x \tag{29}$$

Given that $Y(x, 0) = 1, \quad Y(0, t) = 1$

Writing the above equation in the form

$$2xY_t(x, t) + Y_x(x, t) = 2x \tag{30}$$

$$2x[sy(x, s) - Y(x, 0)] + y_x(x, s) = \frac{2x}{s} \tag{31}$$

$$2x[sy(x, s) - 1] + y_x(x, s) = \frac{2x}{s} \tag{32}$$

$$\frac{dy}{dx} + 2xsy = 2x + \frac{2x}{s} \tag{33}$$

$$= 2x\left(1 + \frac{1}{s}\right) \tag{34}$$

This is linear differential equation of the first order. The integrating factor is

$$e^{\int 2xsdx} = e^{sx^2} \tag{35}$$

$$e^{sx^2} = \int 2x\left(1 + \frac{1}{s}\right)e^{sx^2}dx + c$$

$$= \frac{1}{s}\left(1 + \frac{1}{s}\right)e^{sx^2} + c \tag{36}$$

$$y(x, s) = \frac{1}{s}\left(1 + \frac{1}{s}\right) + c\, e^{-sx^2}$$

$$\frac{1}{s} = \frac{1}{s}\left(1 + \frac{1}{s}\right) + c \tag{37}$$

$$c = -\frac{1}{s^2}$$

$$y(x, s) = \frac{1}{s}\left(1 + \frac{1}{s}\right) - \frac{1}{s^2}\, e^{-sx^2}$$

$$= \frac{1}{s} + \frac{1}{s^2} - \frac{1}{s^2}\, e^{-sx^2}$$

$$Y(x, t) = \begin{cases} 1 + t & 0 \le t \le x^2 \\ 1 + x^2 & t \ge x^2 \end{cases} \tag{38}$$

5 Conclusion

In this paper, we present Alenezi integral transform. we demonstrate the old integral transforms and compared with Alenezi transform. It has demonstrated that Alenezi integral transform accurate than and satisfy the exact solution like Elzaki , Sumudu, and Laplace transforms for various value of m(s) and n(s).

We demonstrate Alenezi transform for the solutions of ODE, and integral equations. Some examples are used to demonstrate the efficiency of this technique.

Competing Interests

Author has declared that no competing interests exist.

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